G 849 (Pages : 2) Reg. No.....................

Name.....................

B.TECH. DEGREE EXAMINATION, MAY 2014

Eighth Semester

Branch : Civil/Mechanical/Electrical and Electronics Engineering/Electronics and Communication Engineering/Computer Science and Engineering/Information Technology

ADVANCED MATHEMATICS (Elective II) (CMELRT)

(Old Scheme—Supplementary/Mercy Chance)

[Prior to 2010 Admissions]

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*
All questions carry equal marks.

1. (a) Convert the boundary values problem to integral equations using Greens Function using the Green's function G (x, t).

Or

(b) Transform the differential $\frac{d^2y}{dx^2} + y = x, \ y(0) = 1 \ \ y(1) = 0$ to a Fredholm Integral equation finding corresponding Green's function.

2. (a) Show that the function $y(x) = xe^x$ is a solution of the integral equation

$$y(x) = \sin x + 2 \int_0^x \cos(x-t) \ y(t)\, dt.$$

Or

(b) Solve the integral equation $\int_0^x y(t)\, y(x-t)\, dt = 4 \sin 9x.$

3. (a) Show that :

(i) $\Gamma n = \int_0^1 \left[\log\left(\frac{1}{y}\right)\right]^{n-1} dy.$

(ii) Prove that $\beta(m, n+1) + \beta(m+1, n) = \beta(m, n).$

(iii) Evaluate $\Gamma\left(\frac{1}{4}\right) \ \Gamma\left(\frac{3}{4}\right).$

Or

Turn over

(b) Prove that $\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \cdot \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi$. Evaluate (i) $\int_0^{\infty} \frac{dx}{1+x^4}$; (ii) $\int_0^{1} \frac{x\, dx}{\sqrt{1+x^5}}$.

4. (a) (i) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomial.

(ii) Prove that $P_n(-x) = (-1)^n P_n(x)$.

Or

(b) Prove that $\left(1 - 2xt + t^2\right)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\, t^n$. Prove that $P_n(0) = \frac{(-1)^n 2n!}{2^{2n} (n!)^2}$.

5. (a) Derive one dimensional heat equation $\frac{\partial y}{\partial t} = e^2 \frac{\partial^2 y}{\partial x^2}$ and solves the heat equation.

Or

(b) A rod of length 'l' has its ends at A and B at 0° C and 100° C respectively untill steady state condition prevail. If the temperature at B is suddenly reduced to 0°C and kept so while that of A is continued to be maintained at 0°C. Prove that $\theta(x, t)$ at distance x from A at any time 't' is given by $\theta(x, t) = \frac{200}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{e}\, e^{-\left(\frac{e^2 n^2 \pi^2}{l^2}\right) t}$ where $\theta(x, t)$ satisfies the equation $\frac{\partial \theta}{\partial t} + e^2 \frac{\partial^2 \theta}{\partial x^2}$.

[5 × 20 = 100 marks]