

## B.TECH. DEGREE EXAMINATION, MAY 2014

## Fourth Semester

## ENGINEERING MATHEMATICS—III (CMELRPTANSUF)

(Old Scheme—Supplementary/Mercy Chance—Prior to 2010 admissions)



Time : Three Hours

Maximum : 100 Marks

*Answer all questions.**Each full question carries 20 marks.**Use of Statistical tables is permitted.*

1. (a) Solve  $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$ . (5 marks)
- (b) Solve  $(D^3 + 1)y = \sin(2x + 3)$ . (7 marks)
- (c) Solve  $(3x + 2)^2 \frac{d^2y}{dx^2} + 3(3x + 2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ . (8 marks)

Or

2. (a) Solve  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x \sin(\log x)$ . (8 marks)
- (b) By method of variation of parameters solve  $y'' - 2y' + 2y = e^x \tan x$ . (7 marks)
- (c) Solve  $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ . (5 marks)

3. (a) From the p.d.e. by eliminating the arbitrary function from  $z = f(x + it) + g(x - it)$ . (5 marks)

- (b) Solve  $px - qz = z^2 + (x + y)^2$ . (7 marks)

- (c) A string is stretched and fastened to two points  $l$  apart motion is started by displacing the

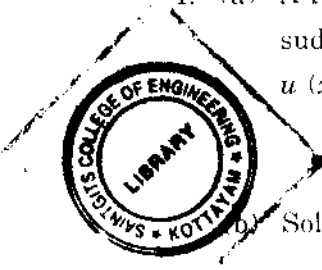
string in the form  $y = a \sin \frac{\pi x}{l}$  from which it is released at time  $t = 0$ . Show that the displacement of any point at a distance  $x$  from one end at time  $t$  is given by

$$y(x, t) = a \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l}$$

(8 marks)

Or

Turn over



4. (a) A rod of length  $l$  with insulated sides is initially at a uniform temperature  $u_0$  its ends are suddenly cooled to  $0^\circ\text{C}$  and are kept at that temperature. Find the temperature function  $u(x,t)$ .

(8 marks)

(b) Solve  $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - \frac{6\partial^2 z}{\partial y^2} = 0$ .

(5 marks)

(c) Solve  $(p^2 + q^2)y = qz$ .

(7 marks)

5. (a) Using Fourier sine integral show that :

$$\int_0^\infty \frac{1 - \cos \pi \lambda}{\lambda} \sin x \lambda \, d\lambda = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

(8 marks)

- (b) Find the Fourier transform of  $f(x) = \begin{cases} 1 - x^2, & \text{of } |x| < 1 \\ 0, & |x| > 1 \end{cases}$  and use it to evaluate

$$\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx$$

(12 marks)

Or

6. (a) Find the Fourier cosine transform of  $e^{-x^2}$ .

(8 marks)

(b) Using Parseval's identity show that  $\int_0^\infty \frac{x^2 dx}{(1+x^2)^2} = \frac{\pi}{4}$ .

(12 marks)

7. (a) Out of 800 families with *four* children each how many families would you expect to have

- (i) 2 boys and 2 girls.                      (ii) Atleast one boy.  
 (iii) No girl.                                      (iv) Atleast 2 girls.

Assume equal probabilities for boys and girls.

(10 marks)

- (b) Derive the mean and variance of Poisson distribution.

(10 marks)

Or



8. (a) Fit a binomial distribution to the following data :

$x:$	0	1	2	3	4	5
	2	14	20	34	22	8

(12 marks)

(b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

(8 marks)

9. (a) The following figures refer to observations in two independent samples :

Sample I : 25 30 28 34 27 20 13 32 22 38

Sample II : 40 34 22 20 31 40 30 23 36 17

Analyse whether the samples have been drawn from the populations of equal mean.

(12 marks)

(b) A coin was tossed 400 times and returned heads 216 times. Test the hypothesis that the coin is unbiased.

(8 marks)

Or

10. (a) Two independent samples of sizes 7 and 6 had the following values :

Sample A : 28 30 32 33 31 29 34

Sample B : 29 30 30 24 27 28

Examine whether the samples have been drawn from normal populations having the same variance.

(12 marks)

(b) A sample of 20 items has mean 42 units and S.D. 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.

(8 marks)

[5 × 20 = 100 marks]