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Reg. No..............................

Name..................................

# B.TECH. DEGREE EXAMINATION, MAY 2016

## Fourth Semester

### ENGINEERING MATHEMATICS – III (CMELRPTANSUF)

(Common to all branches)

[Prior to 2010 Admissions—Supplementary/Mercy Chance]

Time : Three Hours

Maximum : 100 Marks

*Answer **all** questions.*
*Each full question carries 20 marks.*
*Use of Statistical table is permitted.*

1. (a) Solve $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 29y = 0$, given that when $x = 0$, $y = 0$ and $\dfrac{dy}{dx} = 15$.

    (5 marks)

   (b) Solve $\left(D^2 - 1\right)y = x\sin x + \left(1 + x^2\right)e^x$.

    (5 marks)

   (c) A particle of mass of 4 gm executing SHM has velocities 8 cm/sec and 6 cm/sec respectively when it is at distances 3 cm and 4 cm from the centre of its path. Find its period and amplitude. Find also the force acting on the particle when it is at a distance of 1 cm from the centre.

    (10 marks)

*Or*

2. (a) Solve by the method of variation of parameters :

   $y'' - 2y' + 2y = e^x \tan x$.

    (6 marks)

   (b) Solve $x\,dx + y\,dy + z\,dz = 0$, $yz\,dx + zx\,dy + xy\,dz = 0$.

    (6 marks)

   (c) A radial displacement $u$ in a rotating disc at a distance $r$ from the axis is :

   $r^2\dfrac{d^2u}{dr^2} + r\dfrac{du}{dr} - u + kr^3 = 0$, where $k$ is a constant. Solve the equation under the conditions

   $u = 0$ where $r = 0$, $u = 0$ when $r = a$.

    (8 marks)

**Turn over**

3. (a) Find the differential equation of all spheres whose centers lie on the $z$-axis. (6 marks)

   (b) Solve $x(y-z)p + y(z-x)q = z(x-y)$. (6 marks)

   (c) A bar 10 cm long, with insulated sides has its ends A and B maintained at temperatures 50°C and 100°C respectively, until steady-state conditions prevail. The temperature at A is suddenly raised to 90°C and at the same time that at B is lowered to 70°C. Calculate the temperature distribution in the bar at time $t$.

   (8 marks)

*Or*

4. (a) Verify that $z = f\left(x^2 + y^2\right)$ is a solution of $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$. (6 marks)

   (b) Solve $z^2\left(p^2 z^2 + q^2\right) = 1$. (7 marks)

   (c) Solve by Charpit's method $z^2 = pq\,xy$. (7 marks)

5. (a) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\displaystyle\int_0^\infty \frac{\sin x}{x}\,dx$. (12 marks)

   (b) Find the Fourier sine transform of $\dfrac{1}{x\left(x^2 + k^2\right)}$. (8 marks)

*Or*

6. (a) By using Fourier Integral expansion formula, show that :

$$\frac{1}{\pi}\int_0^\infty \frac{\cos\alpha x + \alpha\sin\alpha x}{1+\alpha^2}\,d\alpha = \begin{cases} 0, & x \le 0 \\ e^{-x}, & x > 0. \end{cases}$$

   (8 marks)

   (b) Find the Fourier sine integral for $f(x) = e^{-\beta x}$. Hence show that $\displaystyle\int_0^\infty \frac{\lambda\sin\lambda x}{\lambda^2 + \beta^2}\,d\lambda = \frac{\pi}{2}e^{-\beta x}$.

   (12 marks)

7. (a) In an examination taken by 3000 candidates, the average and the standard deviation of marks obtained (normally distributed) are 40% and 10% respectively. Find :

    (i) How many will pass, if 50% is fixed as pass minimum ?

    (ii) What should be the minimum if 2200 candidates are to pass ?

    (iii) How many have scored more than 75% marks ?

$(3 \times 5 = 15$ marks)

(b) A certain screw making machine produces on average 2 defective screws out of 100, and packs them in boxes of 500. Find the probability that a box contains 15 defective screws ?

(5 marks)

*Or*

8. (a) The probability that a man aged 55 years will die before reaching the age 60 years may be taken as 0.018. One of a group of 400 men, now aged 55 years, what is the probability that two men will die within the next 5 years ?

(8 marks)

(b) The following data are the numbers of seeds germinating out of 10 on damp filter for 80 sets of seeds. Fit a binomial distribution to this data :

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f$ | 5 | 22 | 29 | 16 | 5 | 1 | 1 | 0 | 1 | 1 | 1 | 82 |

(12 marks)

9. (a) The following values gives the lengths of 12 samples of Kanchipuram saries taken from a consignment : 47, 48, 46, 49, 46, 52, 45, 43, 47, 46, 45, 50. Test if the mean length can be taken as 46.
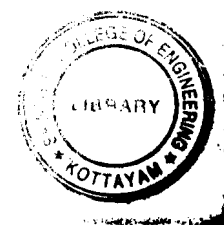
(8 marks)

(b) 500 students at school were graded according to their intelligence and economic conditions of their homes. Examine whether there is any association between economic condition and intelligence, from the following data :

| Economic conditions | Intelligence | |
|---|---|---|
| | Good | Bad |
| Rich | 85 | 75 |
| Poor | 165 | 175 |

(12 marks)

*Or*

**Turn over**

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10. (a) Two random samples reveal the following data :

| Sample No. | Size | Mean | Variance |
|---|---|---|---|
| I | 16 | 440 | 40 |
| II | 25 | 460 | 42 |

Test whether the samples come from the same normal population.

(8 marks

(b) To compare the prices of a certain product in two cities, 10 shops were related at random in each town. The price was noted below :

City 1  : 61  63  56  63  56  63  59  56  44  61

City 2  : 55  54  47  59  51  61  57  54  64  58

Test whether the average prices can be said to be the same in two cities.

(12 marks)

[5 × 20 = 100 marks]