

**B.TECH. DEGREE EXAMINATION, NOVEMBER 2014****Third Semester**

Branch : Common to all Branches except Computer Science and Information Technology

**ENGINEERING MATHEMATICS-II (CMEPLANSUF)**

(Old Scheme—Prior to 2010 admissions)

[Supplementary/Mercy Chance]

Time : Three Hours

Maximum : 100 Marks

*Answer any one full question from each module.**Each full question carries 20 marks.***Module 1**

1. (a) Verify the formula,  $\frac{d}{dt}(\vec{A} \cdot \vec{B}) = \vec{A} \cdot \frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt} \cdot \vec{B}$  for  $\vec{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ ,  $\vec{B} = \sin t\hat{i} - \cos t\hat{j}$ .
- (b) A particle (position vector  $\vec{r}$ ) is moving in a circle with constant angular velocity  $\omega$ . Show by vector methods, that the acceleration is equal to  $-\omega^2\vec{r}$ .
- (c) If  $u = x^2 + y^2 + z^2$  and  $\vec{V} = x\hat{i} + y\hat{j} + z\hat{k}$ , show that  $\operatorname{div}(u\vec{V}) = 5u$ .

*Or*

2. (a) If  $u = x + y + z$ ,  $v = x^2 + y^2 + z^2$ ,  $w = yz + zx + xy$ , prove that :

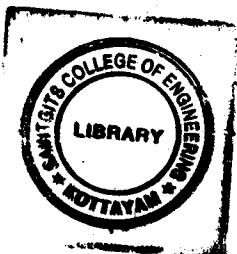
$$(\operatorname{grad} u) \cdot [(\operatorname{grad}(V)) \times (\operatorname{grad}(w))] = 0.$$

- (b) Show that the vector field  $\vec{A}$ , where  $\vec{A} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$  is irrotational, and find the scalar  $\phi$  such that  $A = \operatorname{grad} \phi$ .

**Module 2**

3. (a) Find the work done in moving a particle once round the circle  $x^2 + y^2 = 9$  in the  $x$ - $y$ -plane if the field of force is  $\vec{F} = (2x - y - z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$ .

Turn over



- (b) Show that  $\iint_S \bar{F} \cdot \hat{n} dS = \frac{3}{2}$ , where  $\bar{F} = 4xz \hat{i} - y^2 \hat{j} + yz \hat{k}$  and S is the surface of the cube bounded by the planes  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ .

- (c) Use divergence theorem to show that  $\oint_C r^n \bar{r} \cdot d\bar{S} = (n+3) \int_V r^n dV$  ( $n \neq -3$ ).

*Or*

4. (a) If S is any closed surface enclosing a volume V and  $\bar{F} = x \hat{i} + 2y \hat{j} + 3z \hat{k}$ , prove that

$$\iint_S \bar{F} \cdot \hat{n} dS = 6V$$

- (b) Verify Stoke's theorem for the function  $\bar{F} = x^2 \hat{i} + xy \hat{j}$  integrated round the square whose sides are  $x = 0, y = 0, x = a$  and  $y = a$  in the plane  $z = 0$ .

- (c) The acceleration of a particle at any time  $t$  is given by  $\bar{a} = 12 \cos 2t \hat{i} - 8 \sin 2t \hat{j} + 16t \hat{k}$ . If the velocity  $\bar{v}$  and displacement  $\bar{r}$  are zero at  $t = 0$ , find  $\bar{v}$  and  $\bar{r}$  at any time  $t$ .

### Module 3

5. (a) If  $z_0$  is the upper half of the  $z$ -plane, show that the bilinear transformation  $w = e^{ia} \left( \frac{z-z_0}{z+z_0} \right)$

maps the upper half of the  $z$ -plane into the interior of the unit circle at the origin in the  $w$ -plane.

- (b) Find the analytic function whose real part is  $e^x(x \cos y - y \sin y)$ .

- (c) Show that the transform  $w = z + \frac{(a^2 - b^2)}{4z}$  transforms the circle of radius  $\frac{a+b}{2}$ , centre at the origin, in the  $z$ -plane into ellipse of semi-axes  $a, b$  in the  $w$ -plane.

*Or*

6. (a) If  $f(z)$  is an analytic function prove that  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \log |f'(z)| = 0$ .

(b) If  $w = \phi + i\psi$  represents the complex potential for an electric field and  $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$ ,

determine the function  $\phi$ .

(c) Under the transformation  $w = \frac{z-i}{1-iz}$ , find the map of the circle  $|z| = 1$  in the  $w$ -plane.

#### Module 4

7. (a) Evaluate  $\Delta^2 \cos(cx+d)$ , the interval of differencing being  $h$ .

(b) If  $u_0 = 3, u_1 = 12, u_2 = 81, u_3 = 200, u_4 = 100, u_5 = 8$ , find the value of  $\Delta^5 u_0$ .

(c) A function  $f(x)$  is given by the following table. Find  $f(0.2)$  by a suitable formula :

$x$	:	0	1	2	3	4	5	6
$f(x)$	:	178	183	190	202	218	222	230

Or

8. (a) Use Lagrange's interpolation formula to find the value of  $y$  when  $x = 10$ , if the following table of  $x$  and  $y$  is given :

$x$	:	5	6	9	11
$y$	:	12	13	14	16

(b) Apply Stirling's formula to find  $f(0.42)$  if  $f(0.30) = 0.1179, f(0.35) = 0.1368, f(0.40) = 0.1554, f(0.45) = 0.1736, f(0.50) = 0.1915$ .

#### Module 5

9. (a) The following table gives the values of a function at equal intervals :

$x$	:	0.0	0.5	1.0	1.5	2.0
$f(x)$	:	0.3988	0.3522	0.2421	0.1290	0.0541

Evaluate  $f(1.8), f'(1.5)$  and  $\int_0^2 f(x) dx$ , stating the formula used.

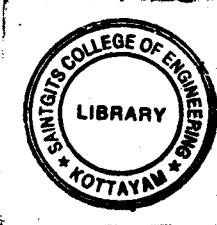
(15 marks)

(b) Solve  $u_{n+2} - 7u_{n+1} + 10u_n = 12e^{3n} + 4^n$ .

(5 marks)

Or

Turn over



10. (a) From the following data :

$x$	:	0.00	0.05	0.10	0.15	0.20	0.25
$y$	:	0.00000	0.10018	0.20132	0.30458	0.41075	0.52110

Evaluate  $\frac{dy}{dx}$  at  $x = 1.00$ .

(b) Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  at  $x = 1.1$  and  $x = 1.6$ .

$x$	:	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$y$	:	7.989	8.413	8.782	9.129	9.452	9.750	10.022

(5 × 20 = 100 marks)

