F 3081 (Pages : 4) Reg. No.................................

Name.................................

B.TECH. DEGREE EXAMINATION, NOVEMBER 2014

Third Semester

Branch : Common to all Branches except Computer Science and Information Technology

ENGINEERING MATHEMATICS—II (CMEPLANSUF)

(Old Scheme—Prior to 2010 admissions)

[Supplementary/Mercy Chance]

Time : Three Hours Maximum : 100 Marks

Answer any **one** *full question from each module.*
Each full question carries 20 *marks.*

Module 1

1. (a) Verify the formula, $\frac{d}{dt}\left(\vec{A}\cdot\vec{B}\right)=\vec{A}\cdot\frac{d\vec{B}}{dt}+\frac{d\vec{A}}{dt}\cdot\vec{B}$ for $\vec{A}=5t^2\hat{i}+t\,\hat{j}-t^3\hat{K},\vec{B}=\sin t\,\hat{i}-\cos t\,\hat{j}$.

   (b) A particle (position vector $\vec{r}$) is moving in a circle with constant angular velocity w. Show by vector methods, that the acceleration is equal to $-w^2\vec{r}$.

   (c) If $u=x^2+y^2+z^2$ and $\vec{V}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$, show that $\operatorname{div}\left(u\vec{V}\right)=5u$.

Or

2. (a) If $u=x+y+z,\ v=x^2+y^2+z^2, w=yz+zx+xy$, prove that :

   $(\text{grad } u)\cdot\left[(\text{grad}(V))\times(\text{grad}(w))\right]=0.$

   (b) Show that the vector field $\vec{A}$, where $\vec{A}=\left(x^2-y^2+x\right)i-(2xy+y)\,\hat{j}$ is irrotational, and find the scalar ϕ such that $A=\text{grad }\phi$.

Module 2

3. (a) Find the work done in moving a particle once round the circle $x^2+y^2=9$ in the x-y-plane if the field of force is $\vec{F}=(2x-y-z)\,\hat{i}+\left(x+y-z^2\right)\hat{j}+(3x-2y+4z)\,\hat{k}$.

Turn over

(b) Show that $\iint_S \vec{F}\cdot\hat{n}\,dS = \frac{3}{2}$, where $\vec{F} = 4xz\,\hat{i} - y^2\hat{j} + yz\,\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(c) Use divergence theorem to show that $\oint_C r^n\vec{r}\cdot d\vec{S} = (n + 3) = \int_V r^n dV\,(n \neq -3)$.

Or

4. (a) If S is any closed surface enclosing a volume V and $\vec{F} = x\,\hat{i} + 2y\,\hat{j} + 3z\,\hat{k}$, prove that $\iint_S \vec{F}\cdot\vec{n}\,dS = 6V$

(b) Verify Stoke's theorem for the function $\vec{F} = x^2\,\hat{i} + xy\,\hat{j}$ integrated round the square whose sides are $x = 0, y = 0, x = a$ and $y = a$ in the plane $z = 0$.

(c) The acceleration of a particle at any time t is given by $\vec{a} = 12\cos 2t\,\hat{i} - 8\sin 2t\,\hat{j} + 16t\,\hat{k}$. If the velocity $\vec{v}$ and displacement $\vec{r}$ are zero at $t = 0$, find $\vec{v}$ and $\vec{r}$ at any time t_i.

Module 3

5. (a) If z_0 is the upper half of the z-plane, show that the bilinear transformation $w = e^{i\alpha}\left(\frac{z - z_0}{z - z_0}\right)$ maps the upper half of the z-plane into the interior of the unit circle at the origin in the w-plane.

(b) Find the analytic function whose real part is $e^x(x\cos y - y\sin y)$.

(c) Show that the transform $w = z + \frac{(a^2 - b^2)}{4z}$ transforms the circle of radius $\frac{a+b}{2}$, centre at the origin, in the z-plane into ellipse of semi-axes a, b in the w-plane.

Or

6. (a) If $f(z)$ is an analytic function prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f'(z)| = 0$.

(b) If $w = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, determine the function ϕ.

(c) Under the transformation $w = \frac{z-i}{1-iz}$, find the map of the circle $|z| = 1$ in the w-plane.

Module 4

7. (a) Evaluate $\Delta^2 \cos(cx + d)$, the interval of differencing being h.

(b) If $u_0 = 3, u_1 = 12, u_2 = 81, u_3 = 200, u_4 = 100, u_5 = 8$, find the value of $\Delta^5 u_0$.

(c) A function $f(x)$ is given by the following table. Find $f(0.2)$ by a suitable formula :

x :	0	1	2	3	4	5	6
$f(x)$:	178	183	190	202	218	222	230

Or

8. (a) Use Lagrange's interpolation formula to find the value of y when $x = 10$, if the following table of x and y is given :

x :	5	6	9	11
y :	12	13	14	16

(b) Apply Stirling's formula to find $f(0.42)$ if $f(0.30) = 0.1179$, $f(0.35) = 0.1368$, $f(0.40) = 0.1554$, $f(0.45) = 0.1736$, $f(0.50) = 0.1915$.

Module 5

9. (a) The following table gives the values of a function at equal intervals :

x :	0.0	0.5	1.0	1.5	2.0
$f(x)$:	0.3988	0.3522	0.2421	0.1290	0.0541

Evaluate $f(1.8)$, $f'(1.5)$ and $\int_0^2 f(x)\,dx$, stating the formula used.

(15 marks)

(b) Solve $u_{n+2} - 7u_{n+1} + 10u_n = 12e^{3n} + 4^n$. (5 marks)

Or

10. (a) From the following data :

x	:	0.00	0.05	0.10	0.15	0.20	0.25
y	:	0.00000	0.10018	0.20132	0.30458	0.41075	0.52110

Evaluate $\frac{dy}{dx}$ at $x = 1.00$.

(b) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$ and $x = 1.6$.

x	:	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	:	7.989	8.413	8.782	9.129	9.452	9.750	10.022

(5 × 20 = 100 marks)