Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

THIRD SEMESTER B.TECH DEGREE EXAMINATION (R,S), DECEMBER 2023
COMPUTER SCIENCE AND ENGINEERING
(2020 SCHEME)

Course Code : **20MAT203**

Course Name: **Discrete Mathematical Structures**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Define Tautology with example.
2. Use Truth table to verify the Rule Modus Ponens.
3. In how many ways can the letters of the word 'MATHEMATICS' be arranged such that vowels must always come together?
4. State Pigeon hole principle.
5. Let f: R→ R defined by f(x) = x^4 -x. Check whether f is one to one function?
6. Let $A = \{2,3,6,12,24,36\}$. Draw the Hasse Diagram of $(A, /)$.
7. Determine the coefficient of x^{50} in $f(x) = (x^7 + x^8 + x^9 \ldots\ldots..)^6$
8. Define a monoid. Give an example.
9. State Lagrange's Theorem.
10. Define Semi group with example.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Check the validity of the following argument by using truth table (7)
$(p \lor q) \land (p \to r) \land (q \to r) \to r$

b) Check whether the propositions $p \lor (q \land r)$ *and* $(p \lor q) \land r$ are logically equivalent (7)

OR

12. a) Establish the validity of the argument

(8)

$p \rightarrow (q \rightarrow r)$
$p \vee \neg s$
q

$\therefore s \longrightarrow r$

b) Check whether $(p \rightarrow q) \wedge [(q \wedge \neg r) \rightarrow (p \vee r)]$ is a tautology (6)

MODULE II

13. a) Determine the number of integral solutions of $x_1 + x_2 + x_3 + x_4 + x_5 < 40$, where $x_i \geq 0$, $1 \leq i \leq 5$. (6)

b) Determine the number of positive integers $1 \leq n \leq 2000$ where n is not divisible by 2,3,5,7. (8)

OR

14. a) A committee of 8 is to be formed from 16 men and 10 women. In how many ways can the committee be formed if (a) there must be 4 men and 4women (b) there should be an even number of women (c) more women than men (d) at least 6 men. (8)

b) Show that in any group of 8 people, at least 2 have birthdays which fall on the same day of the week in any given year (6)

MODULE III

15. a) If R is a relation in the set of integers defined by $R = \{(x, y): x - y \text{ is divisible by } 3\}$. Prove that R is an equivalence relation. Find the distinct equivalent classes of R. (8)

b) Let $< D_{20}, / >$ denote the poset of all divisors of 20. Show that D_{20} is a lattice by using meet join table. (6)

OR

16. a) Let S be a finite set and P(S) be the power set of S, R ={ (A, B) / A⊆B and A, B ∈ P(S). Show that R is a partial order relation in P(S). (6)

b) If f, g, h are functions of integers such that $f(n) = n^2, g(n) = n + 1$ and $h(n) = n - 1$. Find $f \circ g$, $g \circ h$, $(f \circ g) \circ h$ & $g \circ (f \circ h)$. (8)

MODULE IV

17. a) Solve the Recurrence relation $a_{n+1} - 2a_n = 5, n \geq 0, a_0 = 1$ (6)

b) Solve the recurrence relation $a_{n+2} - 10a_{n+1} + 21a_n = 3n - 2$, $n \geq 0, a_0 = 1, a_1 = 0$ (8)

OR

18. a) Solve the Recurrence relation $a_n - 5a_{n-1} - 6a_{n-2} = 0$, $n \geq 2$, $a_0 = 1$, $a_1 = 3$ (6)

b) Solve the Recurrence relation $a_{n+2} + 3a_{n+1} + 2a_n = 3^n, n \geq 0$, $a_0 = 0$ and $a_1 = 1$. (8)

MODULE V

19. a) Show that any group G is abelian if and only if $(ab)^2 = a^2 b^2$ for all a, b in G. (6)

b) Show that Q^+ of all positive rational numbers form an abelian group under the operation * defined by $a*b = \frac{ab}{2}$ where a, b ∈ Q^+. (8)

OR

20. a) Show that $< Z_7^* \, . >$ is an abelian group where "." is the operator "multiplication modulo 7". (6)

b) Verify that the set $\{1, -1, i, -i\}$ is a cyclic group under the operation multiplication. (8)

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