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SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH DEGREE EXAMINATION (Regular), DECEMBER 2023

(Power Systems)

(2021 Scheme)

Course Code : **21PS101**

Course Name: **Applied Mathematics**

Max. Marks : **60** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the Z-transform of n.
2. Solve Euler's equation for the functional $\int_a^b (y^2 + x^2 y')\, dx$.
3. Show that $y(x) = 1 - x$ is a solution of the integral equation $\int_0^x e^{x-t} y(t) dt = x$.
4. Distinguish between Discrete Time Markov chains and Continuous Time Markov chains.
5. Write the normal equations for fitting a curve $y = a + bx + cx^2$ to the given set of points.
6. Explain briefly the Natural cubic spline approximation.
7. Let V be the real vector space of all functions f from R into R and W be the set of all functions f such that $f(x^2) = [f(x)]^2$. Show that W is *not* a subspace of V.
8. Let T be the linear operator on R^3 defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$. Find the matrix of T in the standard ordered basis for R^3.

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. Using the Residue method, solve $y_k + \frac{1}{9} y_{k-2} = \frac{1}{3^k} \cos\frac{k\pi}{2},\ k \geq 0$. (6)

OR

10. Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$. (6)

MODULE II

11. Find the curves on which the functional $\int_0^{\frac{\pi}{2}} (y'^2 - y^2 + 2xy) dx$ with $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$ be extremized. (6)

OR

12. A particle is moving with a force perpendicular to and proportional to its distance from the line of zero velocity. Show that the path of quickest descent is a circle. (6)

MODULE III

13. Convert the differential equation $y''(x) + xy(x) = 1, y(0) = y'(0) = 0$ to the Volterra integral equation. (6)

OR

14. Solve the Fredholm integral equation $y(x) = 1 + \lambda \int_0^x xty(t)dt$ using successive approximation. (6)

MODULE IV

15. Let $X_1, X_2, \dots, X_n$ be a random sample taken from a population with mean μ and variance σ^2. Show that sample mean $\bar{X}$ and sample variance S^2 are unbiased estimators of μ and σ^2 respectively. (6)

OR

16. Find the Maximum Likelihood Estimate (MLE) of λ, based on random samples taken from Poisson population with parameter λ. (6)

MODULE V

17. Fit a straight line to the following data (6)

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

OR

18. Fit a second-degree parabola of the form $y = a + bx + cx^2$ to the following data. $y(0) = 1.2, y(1) = 1.7, y(2) = 2.1, y(3) = 2.8, y(4) = 5.9$. (6)

MODULE VI

19. Find a basis and dimension for the subspace of R^4 spanned by the four vectors $(1, 0, 2, 1), (2, 1, 3, 1), (1, 1, -1, 0), (2, 1, 1, 3)$. (6)

OR

20. Let T be a linear operator on R^3, the matrix of which in the standard ordered basis is $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$. Find a basis for the range of T and a basis for the null space of T. (6)

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