Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH DEGREE EXAMINATION (Regular), DECEMBER 2023

ROBOTICS AND AUTOMATION

(2021 Scheme)

Course Code: **21RA101**

Course Name: **Advanced Mathematics and Optimization Techniques**

Max. Marks: **60** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Determine whether the set of all 2×2 real matrices is a vector space under regular scalar multiplication but with vector addition defined to be matrix multiplication.

2. Determine whether the transformation T is linear if $T: R^2 \longrightarrow R^2$ is defined by $T[a \quad b] = [a \quad -b]$ for all real numbers a and b.

3. Show that the vectors $\{x, y, z\}$ in R^3 defined by $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $y = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $z = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ are an orthogonal set of vectors.
4. A company manufactures two types of products P1 and P2. Each product uses lathe and milling machine. The processing time per unit of P1 on the lathe is 5 hrs and on the milling machine is 4 hrs. The processing time per unit of P2 on the lathe is 10 hrs and on the milling machine is 4 hrs. The maximum number of hours per week on lathe and milling machine are 60 hrs and 40 hrs respectively. Also the profit per unit of selling P1 and P2 are Rs.6 and Rs.8 respectively. Formulate a LP model to determine the production volume of each of the products such that the total profit is maximized
5. Distinguish between integer programming problem and linear programming problem.
6. Solve by graphical method

   maximize $z = 3x_1 + 4x_2$

   $subject\ to\ x_1 + 2x_2 \leq 4$

   $3x_1 + 2x_2 \leq 6$

   $x_1, x_2 \geq 0$

7. State Kuhn-Tucker conditions for a nonlinear programming problem having a maximization objective function.
8. List and explain the basic assumptions of linear programming problem.

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. Determine whether the set $\left\{\begin{bmatrix}4\\5\\1\end{bmatrix}, \begin{bmatrix}3\\0\\2\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$ is linearly independent or not. (6)

OR

10. Find the coordinate representation for the vector $v = 4t^2 + 3t + 2$ in P^2 with respect to the basis $C = \{t^2 + t\,, t + 1\,, t - 1\}$. (6)

MODULE II

11. Find the transition matrix between the bases $C = \left\{\begin{bmatrix}-1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$ and $D = \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\right\}$ in R^2 and verify $v_D = P_C^D v_C$ for the coordinate representations of $v = \begin{bmatrix}7\\2\end{bmatrix}$ with respect to each basis. (6)

OR

12. Find the matrix representation for the linear transformation $T: M_{2\times2} \longrightarrow M_{2\times2}$ defined by $T\begin{bmatrix}a & b\\c & d\end{bmatrix} = \begin{bmatrix}a + 2b + 3c & 2b - 3c + 4d\\3a - 4b - 5d & 0\end{bmatrix}$ with respect to the standard basis $B = \left\{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\right\}$ (6)

MODULE III

13. Construct a QR decomposition for $A = \begin{bmatrix}1 & 0 & 1\\1 & 1 & 0\\0 & 1 & 1\end{bmatrix}$. (6)

OR

14. Find the least-squares solution to (6)

$$\begin{aligned}1x + 3y &= 180\\2x + 5y &= 100\\5x - 2y &= 60\\-x + 8y &= 130\\10x - y &= 150.\end{aligned}$$

MODULE IV

15. Solve using simplex method (6)
$Maximize\ z = 7x_1 + 6x_2$
$subject\ to\ x_1 + x_2 \leq 4$
$2x_1 + x_2 \leq 6$
$x_1, x_2 \geq 0$

OR

16. Solve by Big M method (6)
$Minimize\ z = 5x_1 + 6x_2$
$Subject\ to\ 2x_1 + 5x_2 \geq 1500$
$3x_1 + x_2 \geq 1200$
$x_1, x_2 \geq 0$

MODULE V

17. Solve the following integer programming problem using branch and bound method. (6)
$Maximize\ z = x_1 + 4x_2$
$subject\ to\ 2x_1 + 4x_2 \leq 7$
$5x_1 + 3x_2 \leq 15$
$x_1, x_2 \geq 0\ and\ are\ integers.$

OR

18. Find the optimum integer solution for the following linear programming problem using Gomory's cutting plane method. (6)
$Maximize\ z = x_1 + x_2$
$Subject\ to\ 3x_1 + 2x_2 \leq 5$
$x_2 \leq 2$
$x_1, x_2 \geq 0\ and\ are\ integers.$

MODULE VI

19. Solve the non linear programming problem using Lagrangian method (6)
Maximize $z = 4x_1 - 0.02x_1^2 + x_2 - 0.02x_2^2$
$Subject\ to\ x_1 + 2x_2 = 120$
$x_1, x_2 \geq 0$

OR

20. Solve the following using Kuhn-Tucker conditions (6)
$Maximise\ z = 3x_1^2 + 14x_1x_2 - 8x_2^2$
$Subject\ to\ 3x_1 + 6x_2 \leq 72$
$x_1, x_2 \geq 0$

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