Register No.: ……………………………… Name: ………………………………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER B.TECH DEGREE EXAMINATION (R,S), DECEMBER 2023

COMMON TO ALL BRANCHES

(2020 SCHEME)

Course Code : **20MAT101**

Course Name: **Linear Algebra and Calculus**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. If 1 is an eigen value of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, without using the characteristic equation find the other eigen values. Also find the eigen values of A^3.
2. Determine the rank of the matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$
3. Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the $y -$ direction at the point $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$.
4. Given $f = e^x \sin y$, show that the function satisfies the Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
5. Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{(1+x^2)(1+y^2)}$.
6. Use polar co-ordinates to evaluate $\iint e^{-(x^2+y^2)} dA$ when R is the region enclosed by the circle $x^2 + y^2 = 1$.
7. Determine whether the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$ converges, if it converges find the sum.
8. Using limit comparison test, determine whether $\sum_{k=1}^{\infty} \frac{1}{2k^2+k}$.
9. Obtain the Maclaurin series expansion of $f(x) = \sin x$.
10. Find the Fourier coefficients $a_0, a_n,$ and b_n of the function $f(x) = x^2, -\pi < x < \pi$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Using Gauss elimination method solve the following system of equations, (7)

$$x + 2y - z + w = 6$$
$$-x + y + 2z - w = 3$$
$$2x - y + 2z + 2w = 14$$
$$x + y - z + 2z = 8$$

b) Obtain the eigen values and eigen vectors of $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ (7)

OR

12. a) Diagonalize the matrix, $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (7)

b) Transform $17x_1^2 - 30x_1x_2 + 17x_2^2$ into the principal axis form using orthogonal transformation. (7)

MODULE II

13. a) If $u = f\left(\frac{x}{y}, \frac{y}{x}, \frac{z}{x}\right)$, Find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (7)

b) Find the local linear approximation $L(x, y)$ to $f(x, y) = \frac{1}{\sqrt{x^2+y^2}}$ at the point $P(4,3)$. Also compare the error in approximating $f(x, y)$ by $L(x, y)$ at $Q(3.92, 3.01)$ with the distance between P and Q (7)

OR

14. a) Locate all relative extrema and saddle points if any of $f(x, y) = xy - x^3 - y^2$ (7)

b) The length and breadth of a rectangle are measured with errors of at most 4% and 5% respectively. Use differentials to approximate the maximum percentage error in the calculated area. (7)

MODULE III

15. a) Use a double integral to evaluate the area bounded by the $x -$ axis, $y = 2x$ and $x + y = 1$ (7)

b) Use cylindrical co-ordinates to find the volume of the solid that is bounded above and below by the sphere $x^2 + y^2 + z^2 = 9$ and inside the cylinder $x^2 + y^2 = 4$ (7)

OR

16. a) Evaluate $\int_0^1 \int_x^1 e^{y^2} dx\, dy$ after changing the order of integration. (7)

b) Use triple integral to find the volume of the solid in the first octant bounded by the coordinate planes and the plane $3x + 6y + 4z = 12$ (7)

MODULE IV

17. a) (i) Check the convergence of $\sum_{k=1}^{\infty} cot^{-1}(k^2)$

(ii) Determine whether the series $1 - \frac{1}{2} + \frac{1}{3} - \ldots$ converges. (7)

b) Find the sum of the series $\sum_{k=1}^{\infty}\left(\frac{3}{4^k} - \frac{2}{5^{k-1}}\right)$ (7)

OR

18. a) (i) Check the convergence of the series, $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+3}{k(k+1)}$

(ii) Determine whether the series $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k+7}}$ (7)

b) Examine the convergence of $\sum_{k=0}^{\infty}\frac{(k+1)!}{4!k!4^k}$ (7)

MODULE V

19. a) Obtain the Maclaurin series expansion of $tan^{-1} x$ and hence express π as an infinite series. (7)

b) Find the half range cosine series of $f(x) = \begin{cases} 0 & -2 < x < -1 \\ k & -1 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$ (7)

OR

20. a) Find the binomial series for $f(x) = \frac{1}{\sqrt{1+x}}$ (7)

b) Obtain the Fourier series of $f(x) = x - x^2$, for $-\pi \le x \le \pi$. Hence deduce the value of $1 - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$ (7)

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