

Register No.: Name:

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B. TECH DEGREE EXAMINATION (S), AUGUST 2023**COMMON TO CH,CE,FT,ME,RE****(2020 SCHEME)****Course Code: 20MAT202****Course Name: Probability, Statistics and Numerical Methods****Max. Marks: 100****Duration: 3 Hours****PART A****(Answer all questions. Each question carries 3 marks)**

1. If X is any random variable with $E(X)=2$ and $E[X(X-1)] = 6$, find $\text{Var}(X)$?
2. The mean and variance of a binomial random variable X are 16 and 8 respectively. Find $P(X=0)$?
3. For the distribution with PDF $f(x) = \begin{cases} kx(2-x), & 0 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$.

Find the value of k?

4. If X has uniform distribution in $(-3,3)$, find $P(|X-2| < 2)$?
5. A sample of 100 items gave a mean 7.4kg and a standard deviation 1.2kg. Find 95% confidence interval for population mean?
6. Explain the Types of errors in hypothesis testing?
7. Use Newton's forward difference formula to find the interpolating polynomial for the following data:

x	0	1	2	3
y	0	2	6	18

8. A river is 80m wide, the depth d in meters at a distance x meters from one bank is given by the following table. Calculate the area of cross section of the river using Simpson's rule.

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

9. Use Euler's method to solve $\frac{dy}{dx} = x + xy + y$, $y(0) = 1$. Compute y at $x = 0.15$ by taking $h = 0.05$.
10. Write the normal equations to fit a parabola by the method of least squares for a given data.

PART B*(Answer one full question from each module, each question carries 14 marks)***MODULE I**

11. a) Out of 1000 families each with 4 children chosen at random how many would you expect to have (i) at least one boy (ii) 1 or 2 girls (iii) no girls? (7)
- b) Assuming that the probability of a fatal accident in a factory during the year is $\frac{1}{1200}$. Calculate the probability that in a factory employing 300 workers, there will be at least two fatal accidents during a year? (7)

OR

12. a) Find α and β if $Y = \alpha X + \beta$ has mean 4 and variance 16 where X is a random variable with mean 8 and variance 4? (7)
- b) Show that the mean and variance of Poisson distribution are the same. (7)

MODULE II

13. a) Let X be a continuous random variable with mean $\mu = 4.35$ and $\sigma = 0.59$. if X follows normal distribution, find (7)
- (i) $P(4 < X < 5)$
- (ii) $P(X > 5.5)$
- b) Let X be a continuous random variable with $f(x) = \begin{cases} 6x(1-x), & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$. Check whether f(x) is a PDF. (7)
- Determine value of b such that $P(x < b) = P(x > b)$.

OR

14. a) A random variable follows exponential distribution with PDF

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find the probability that it will take on a value

- (i) between 1 and 3 (7)
- (ii) greater than 0.5
- (iii) mean and variance
- (iv) the distribution functions

- b) The weekly wages of 1000 workers are normally distributed about a mean of Rs. 500 with a standard deviation of 50. Estimate the number of workers whose weekly wages will be (i) between Rs. 400 and Rs. 600 (ii) less than Rs. 400 (iii) more than Rs. 600 (7)

MODULE III

15. a) A sample of 400 male students is found to have a mean height of 160cm. Can it be reasonably regarded as a sample from a population with mean height 162.5cm and standard deviation 4.5cm? (7)
- b) In a survey conducted to assess what percentage of electorates would support a particular candidate in the forthcoming elections, it was found that 87 of a random sample of 250 voters supported the candidate. To increase the candidate's chances a vigorous campaign was undertaken during the following week. In a sample survey conducted after the campaign 92 of 200 voters surveyed supported the candidate. Test at 5% level of significance, whether the proportion of voters supporting candidate has increased due to the campaign. (7)

OR

16. a) Batteries manufactured at two factories A and B are supposed to be identical. Recently there has been complaints that the batteries from factory B do not last as long as batteries from factory A. To check this, random samples of 50 batteries made at factory A and 60 made at factory B were tested and the sample means and variances of the lifetimes of the batteries were calculated with the following results. (7)

	Sample size	Sample mean \bar{X} (hours)	Sample variance(s^2)
Factory A	50	42.75	1.98
Factory B	60	42.15	1.82

Test at 5% level of significance, whether there is any substance in the complaints?

- b) A die was thrown 9000 times and of these 3220 yielded 3 or 4. Can the die be regarded as unbiased? (7)

MODULE IV

17. a) Using Newton's Raphson's Method, compute a real root of $f(x) = x^3 - 2x - 5$ correct to five decimal places? (7)
- b) Solve the equation $xe^x = 2$ by method of false position? (7)

OR

18. a) Apply Lagrange's interpolation formula to find the value of y when $x = 10$ from the following table:

x	5	6	9	11
y	12	13	14	16

(7)

- b) Using Newton's divided difference interpolation formula evaluate $f(8)$ and $f(15)$ from the following data.

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

(7)

MODULE V

19. a) Solve the following system correct to three decimals using Gauss-Seidel iteration method

$$28x + 4y - z = 32 \quad (7)$$

$$2x + 17y + 4z = 35$$

$$x + 3y + 10z = 24$$

- b) Fit a curve of the form $y = ax + b$ to the following data:

x	1	2	3	4	6	8
y	2.4	3.1	3.5	4.2	5	6

(7)

OR

20. a) Solve y for $x = 1.1$ using Runge-Kutta fourth order method, given

$$\frac{dy}{dx} = x^2 + y^2 \text{ and } y = 1.5 \text{ at } x = 1. \quad (7)$$

- b) Using Adam's Moulton Predictor corrector method solve

$$\frac{dy}{dx} = x^2(1 + y) \text{ for } x = 1.4, \text{ given } y(1)=1, y(1.1)= 1.233, y(1.2)= 1.548, \quad (7)$$

$$y(1.3) = 1.979.$$
