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SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (R), MAY 2023

COMMON TO EE,EC

(2020 SCHEME)

Course Code: **20MAT204**

Course Name: **Probability, Random Processes and Numerical Methods**

Max. Marks: **100** **Duration: 3 Hours**

Scientific calculator and statistical table are allowed.

PART A

(Answer all questions. Each question carries 3 marks)

1. X follows a Binomial distribution such that mean, $E[X] = 2$ and variance, $Var[X] = \frac{4}{3}$. Find $P[X = 5]$.
2. If X is a Poisson random variable such that $P[X = 1] = P[X = 2]$, find the mean and variance of X.
3. If random variable X has uniform distribution in $(-3,3)$, find $P[|X - 2| < 2]$.
4. Find c for which $f(x) = cxe^{-x}, 0 < x < \infty$ is a valid probability density function of a continuous random variable.
5. Define wide sense stationary random process.
6. Determine the variance of the random process $X(t)$ whose autocorrelation function is given by $R_{XX}(\zeta) = 16 + \frac{9}{1+6\zeta^2}$
7. Evaluate the integral $\int_0^1 \frac{x^2}{1+x^3} dx$ using Simpson's 1/3 rule (take h=0.25).
8. Use Newton Raphson method to compute a real root of $f(x) = e^{2x} - x - 6$ lying between 0 and 1.
9. Using Euler's method, find y at $x = 0.25$, given $y' = 2xy, y(0) = 1, h = 0.25$.
10. Use Runge-Kutta method of second order to find $y(0.2)$ for $\frac{dy}{dx} = y + e^x$, $y(0) = 0$ (Take $h = 0.2$)

PART B

(Answer one full question from each module, each question carries 14marks)

MODULE I

11. a) A Random variable X has the following probability distribution

x	1	2	3	4	5	6	7
f(x)	k	$2\,k$	$2\,k$	$3\,k$	k^2	$2k^2$	$7k^2 + k$

Compute (i) k

(ii) $P[0 < X < 5]$

(iii) $P[X > 5]$ (7)

b) Show that the mean and variance of Poisson distribution are the same. (7)

OR

12. a) Prove that a binomial distribution can be approximated to Poisson distribution when n is large, p is small and $np = \lambda$. (7)

b) Let X be a number on a die when thrown. Find the mean and variance of X. (7)

MODULE II

13. a) A random variable is uniformly distributed in $(-\alpha, \alpha), \alpha > 0$. Find the value of α if $P(|X| < 1) = P(|X|) > 1$. (7)

b) In a competitive examination, 5000 students have appeared for engineering mathematics. Their average marks were 62 and standard deviation is 12.If there are only 100 vacancies, find the minimum marks one should score in order to get selected. (7)

OR

14. a) Let X be a continuous random variable with mean $\mu = 4.35$ and $\sigma = 0.59$. If X follows normal distribution, find
$(i) P(4 < X < 5)$
$(ii) P(X > 5.5)$ (7)

b) Derive the formula for mean and variance of uniform distribution (7)

MODULE III

15. a) If $X(t) = A\,cos\omega t + B\,sin\omega t$ where A and B are independent random variables with zero mean and equal variance, show that $X(t)$ is WSS. (7)

b) Show that the random process $X(t) = Acos(\omega t + \theta)$ is WSS if A and ω are constants and θ is a uniformly distributed random variable in $(0,2\pi)$ (7)

OR

16. a) Let $X(t) = Acos(\omega t + \theta)$, where ω is a constant, A and θ are independent and uniform random variables over $(-k, k)$ and $(-\pi, \pi)$ respectively.Find the (7)
   i) Mean of $X(t)$
   ii) Autocorrelation function of $X(t)$

b) A WSS random process $X(t)$ has autocorrelation $R_{xx}(\tau) = 36 + \frac{4}{1+\tau^2}$. Determine the mean, variance and power of the process. (7)

MODULE IV

17. a) Using Lagrange's formula, find the polynomial $f(x)$ to the data $f(1) = 4, f(2) = 5, f(7) = 5, f(8) = 4$. (7)

b) Find the polynomial interpolating the following data using Newton's backward interpolation formula. (7)

x	3	4	5	6	7
y	7	11	16	22	29

OR

18. a) Use Trapezoidal rule to evaluate (7)

i) $\int_0^{\pi/2} \cos x \; dx$ with $n = 6$

ii) $\int_0^6 \frac{1}{1+x^2} dx$ with $n = 6$

b) Using Newton's divided difference interpolation formula, evaluate $f(8)$ and $f(15)$ from the following data: (7)

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

MODULE V

19. a) Using Runge-Kutta method, evaluate $y(0.2)$ for the differential equation (7)

$\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$. (Take h = 0.2)

b) Consider the initial value problem $\frac{dy}{dx} = \frac{x+y}{2}$; $y(0) = 2, y(0.5) = 2.636, y(1) = 3.595, y(1.5) = 4.968$. Compute $y(2)$ by Adams method. (7)

OR

20. a) Use Gauss-Seidel method to solve the following system of equations (7)

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 36$$

b) Use the method of least squares to fit an equation of the form $\mathrm{y} = \mathrm{ax} + \mathrm{b}$ to the following data (7)

$\mathrm{y}(1) = 2.4,\ \mathrm{y}(2) = 3,\ \mathrm{y}(3) = 3.6,\ \mathrm{y}(4) = 4,\ \mathrm{y}(6) = 5,\ \mathrm{y}(8) = 6.$

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