Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (R), MAY 2023

(2020 SCHEME)

Course Code : **20CST284**

Course Name: **Mathematics for Machine Learning**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. If $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 2 & 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ -1 & 0\\ 2 & -1\end{bmatrix}$ form the product of AB. Is BA defined?
2. Check whether the set S = {(1, 1, 0), (1, 0, 1), (0, 1, 1)} is linearly independent in $V_3(R)$.
3. If $\alpha=(2,1,-1,-2)$ and $\beta=(1,3,-1,4)$ then find the value of $\langle\alpha, \beta\rangle$, $\|-3\beta\|$, $\|\alpha+\beta\|$.
4. If $\begin{vmatrix}a & a^2 & a^3-1\\ b & b^2 & b^3-1\\ c & c^2 & c^3-1\end{vmatrix}=0$ in which a, b, c are different, show that abc = 1.
5. Find $\frac{du}{dt}$ when $u=\sin(\frac{x}{y}),\ x=e^t,\ y=t^2$.
6. Define Jacobian of the transformation from *x, y, z to u, v, w.*
7. State Baye's Theorem.
8. If A and B are two independent events, find P(B), when P(AUB) = 0.60 and P(A) = 0.35.
9. Construct the Lagrangian function, for the given non-linear programming problem,

   Max. $Z = 2x_1^2 + x_2^2 + 5x_1x_2$,

   Subject to constraints $4x_1+6x_2=8,\ \ 3x_1-6x_2=1$, and $x_1, x_2 \geq 0$
10. What do you mean by a general Linear Programming Problem and give the matrix form of representing a general Linear Programming Problem

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Solve the following equations by method of matrix inversion (6)

$3x+y+2z=3,\ 2x-3y-z=-3\ \ and\ \ x+2y+z=4$

b) Let V = P_2 and let A = $\{1, 1+x, 1+x+x^2\}$ and B = $\{2+x+x^2,\ x+x^2,\ x\}$. Find the Change of basis matrix from A to B and by using the above also find B to A. (8)

OR

12. a) Find the values of k for which the system of equations $(3k-8)x+3y+3z=0,\ \ 3x+(3k-8)y+3z=0,\ 3x+3y+(3k-8)z=0$ has a non-trivial solution. (6)

b) Let V = $\{a+b\sqrt{2}\ /\ a,b\in Q\}$, Prove that V is a Vector space over Q under usual addition and multiplication. (8)

MODULE II

13. Applying Gram-Schmidt process, Let V be the set of all polynomials of degree $\leq$ 2 together with the zero polynomial. V is a real inner product space with the inner product defined by $<f,g>=\int_{-1}^{1} f(x)g(x)\,dx$, starting with the basis $\{1,x,x^2\}$. Obtain an orthonormal basis for V. (14)

OR

14. a) Find the Eigen values and Eigen vectors of the matrix (6)

$$A=\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

b) Find the singular value decomposition of that matrix $A=\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ (8)

MODULE III

15. a) Show that the functions $u=x+y-z, v=x-y+z,\ w=x^2+y^2+z^2-2yz$ are dependent. Find the relation between them. (8)

b) Explain about the concept "Gradients in a Deep Network". (6)

OR

16. a) Using Taylor's series expand $f(x,y) = x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$ up to 3rd degree terms. (8)

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right]^2 u = \frac{-9}{(x+y+z)^2}.$$ (6)

MODULE IV

17. a) The sales of a stores on a randomly selected day are X thousand dollars, where X is a random variable with a distribution function of the following form,

$$F(x)=\begin{cases} 0, & x<0 \\ \frac{x^2}{2}, & 0 \le x \le 1 \\ k(4x-x^2), & 1 \le x \le 2 \\ 1, & x \ge 2 \end{cases}$$ (8)

Suppose that these stores total sales on any given days are less than 2000 dollars. i) Find k, ii) Let A and B be the events such that the stores total sales are between 500 and 1500 dollars and over 1000 dollars resp. Find the P(A) and P(B), iii) Are A and B independent events?

b) The joint p.d.f of X and Y is given in the following table. Find the a) marginal probability distributions of X and Y, b) Conditional distributions of X given Y and c) Find P [X≤ 4, Y ≤ 3]. (6)

X \ Y	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

OR

18. a) A Discrete random variable X has the following probability distributions: (8)

X	0	1	2	3	4	5	6	7	8
P(X=x)	a	3a	5a	7a	9a	11a	13a	15a	17a

i)Find the value of a, ii) Find P(X < 3), P(0 < X < 3), iii) Find F(x),

iv) Mean and Variance.

b) A bag P contains 3 white and 4 red balls and a bag Q contains 5 white and 7 red balls. One ball is drawn at random from a bag selected at random. If the ball drawn is found to be red, find the chance that it is drawn from the bag Q. (6)

MODULE V

19. Apply Lagrangian Multiplier method and Solve the non-linear programming problem,

Optimize $Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$, (14)

Subject to, $x_1 + x_2 + x_3 = 15$, $2x_1 - x_2 + 2x_3 = 20$, and $x_1, x_2, x_3 \geq 0$

OR

20. Solve by Two-Phase simplex method.

Max. $Z = 2x_1 + x_2 + \frac{1}{4}x_3$,

Subject to constraints $4x_1 + 6x_2 + 3x_3 \leq 8$, (14)

$3x_1 - 6x_2 - 4x_3 \leq 1$,

$2x_1 + 3x_2 - 5x_3 \geq 4$, and $x_1, x_2, x_3 \geq 0$

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