Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (R), MAY 2023

(2020 SCHEME)

Course Code : **20CST294**

Course Name: **Computational Fundamentals for Machine Learning**

Max. Marks : 100 **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Define Basis and Dimension of a vector space
2. Check whether the polynomials x(t) = 1 - t +3 t^2 ; y(t) = 2 + 3t +t^2 ; z(t) = 3 + 4t+3t^2 over R are linearly independent or not.
3. Determine whether the vectors, X = $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and Y = $\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$ are orthogonal or not
4. Find the eigen values and corresponding eigen vectors for A = $\begin{bmatrix} 8 & 4 \\ 2 & 6 \end{bmatrix}$
5. Find the gradient and its magnitude $f(x,y) = \sqrt{x^2 + y^2}$ at (1,2)
6. Expand $f(x) = e^{3x}$ as a Maclaurin series
7. Write note on Bayes' theorem.
8. Find the probability density function of getting Heads for a discrete random experiment of throwing two unbiased coins
9. Find the maximum and minimum values of $f(x,y) = 4x + 4y - y^2 - x^2$ subject to $x^2 + y^2 = 2$
10. Explain the process of Gradient Descent

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Find $Ker(\emptyset), ran(\emptyset)$ and its dimension, where $\emptyset : R^4 \to R^2$ is a linear transformation defined by $\emptyset X = AX$, $A_\emptyset = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ is the transformation matrix (9)

b) Solve by Gauss elimination method (5)

x + y + z = 2

y + z = −2

4y + 6z = −12

OR

12. a) Show that set of real numbers under normal addition and multiplication is a Vector space (7)

b) Solve the following linear system of equations (7)

$y + z - 2w = 0$

$2x - 3y - 3z + 6w = 2$

$4x + y + z - 2w = 4$

MODULE II

13. a) Find the Singular value decomposition of $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ (10)

b) Use Cholesky decomposition to decompose $A = \begin{bmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{bmatrix}$ (4)

OR

14. a) Find the Eigen decomposition of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (10)

b) Using Gram-Schmidt Orthogonalization to orthogonalize the vectors $\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ (4)

MODULE III

15. a) Find the local linear approximation L of $f(x, y) = \sqrt{x^2 + y^2}$ at the point $P(3, 4)$. Compute the error in approximation f by L at the point $Q(3.04, 3.98)$ (7)

b) Locate all relative extrema and saddle points of $f(x, y) = xy - y^2 - x^3 + xy$ (7)

OR

16. a) Expand $f(x,y) = e^{xy}\ at\ (1,1)$,using Taylor's theorem (10)

b) Find the direction of greatest increase of the function $f(x,y) = 4x^2 + y^2 + 2y$ at the point P(1,2). (4)

MODULE IV

17. a) A discrete random variable X has the following probability distribution (7)

X	0	1	2	3
P[X=x]	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k+1}{3}$	$\frac{2k+1}{6}$

Find (1) the value of k

(2) P[X ≤ 2]

(3) Mean

b) In a large consignment of electric bulbs 10% are known to be defective. A random sample of 20 is taken for inspection. Find the probability that (7)

(1) all are good bulbs
(2) exactly 3 are defective bulbs

OR

18. a) Consider the following bivariate distribution of two discrete random variables X and Y,

$$f(x, y) = k(2x + 3y); x = 0,1,2 \text{ and } y = 1,2,3$$

(1) Find the value of k (10)
(2) Compute the marginal probability of x and y
(3) P(X+ Y > 3)

b) Differentiate between independent events and mutually exclusive events in probability, with an example (4)

MODULE V

19. a) Solve the linear programming problem graphically (7)
Max $Z = 18x + 10y$
Subject to constraints:

$$4x + y \leq 20, \ 2x + 3y \leq 30, x \leq 5, y \leq 10 \ x, y \geq 0$$

b) Explain the process of Steepest descent method (7)

OR

20. a) Solve L.P.P using simplex method (8)
Max $Z = 7x + 5y$
Subject to constraints:

$$x + 2y \leq 6, 4x + 3y \leq 12, x, y \geq 0$$

b) Write the Lagrange dual of (6)
Min Z = -5x-3y
Subject to constraints:

$$2x + 2y \leq 33, 2x - 4y \leq 8, -2x + y \leq 5, -x \leq -1, y \leq 8$$

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