Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

THIRD SEMESTER B.TECH DEGREE EXAMINATION (S), FEBRUARY 2023
COMMON TO CE,CH,EC,EE,FT,ME,RA
(2020 SCHEME)

Course Code : **20MAT201**

Course Name: **Partial Differential Equations and Complex Analysis**

Max. Marks : 100 **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Form the partial differential equation for the relation $z = f(x^2 - y^2)$.
2. Solve $\frac{y^2 z}{x} p + xzq = y^2$.
3. Write down the three possible solutions of one-dimensional wave equation.
4. Write the boundary condition and initial condition of the string of length l which is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = \mu_0 \sin\left(\frac{\pi x}{l}\right), 0 < x < l$.
5. Check whether the function $e^x(\cos y - i \sin y)$ is analytic or not.
6. If u and v are real and imaginary part of an analytic function, show that u and v are harmonic.
7. Evaluate $\int \bar{z}\, dz$ over the circle $|z| = 1$.
8. Find the Taylor's series expansion of $f(z) = \sin z$ about z=0.
9. Determine the location and nature of singularity of the function $\frac{e^{-z^2}}{z^2}$.
10. Find the Residue of $\frac{1}{(z+1)^3}$ at its poles.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Find the partial differential equation of all spheres of fixed radius whose center lie on the z axis. (7)

    b) Solve $(y - z)p + (x - y)q = (z - x)$. (7)

OR

12. a) Solve $px^2 + qy^2 = (x + y)z$. (7)

b) Solve by the method of separation of variables $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \;\; where \;\; u(x,0) = 6e^{-3x}$ (7)

MODULE II

13. a) Derive the solution of one-dimensional wave equation. (7)

b) Find the temperature distribution in a rod of length 2m whose end points are maintained at temperature zero and the initial temperature is $f(x) = 100(2x - x^2)$. (7)

OR

14. a) A tightly stretched string with fixed end points x = 0and x = l is initially at rest in its equilibrium position. If each of its points is given velocity $\lambda x(l - x)$.Find the displacement of the string at any distance x from one end at any time t. (7)

b) A rod of length L is heated in such a way that its ends A and B are at zero temperature. If initially its temperature is given by $u = \frac{cx(L-x)}{L^2}$, $0 \le x \le L$,find the temperature at time t. (7)

MODULE III

15. a) If f(z)= u+iv is an analytic function and uv = a constant, show that f(z) is a constant. (7)

b) Verify that $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate v. (7)

OR

16. a) Check whether $f(z) = \log z$ is analytic or not. (7)

b) Determine the region of the w-plane into which the triangular region bounded by $x = 1, y = 1$ and $x + y = 1$ is mapped by $w = z^2$ (7)

MODULE IV

17. a) Evaluate $\int_C z^2 dz$ where C is given by the line $2y = x$ from (0,0) to (2,1). (7)

b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 2$. (7)

OR

18. a) Using Cauchy's integral formula, evaluate $\int_C \frac{2z+1}{z^2+z} dz$ where C is the circle $|z| = 2$. (7)

b) Find the Taylor series expansion of $f(z) = \frac{1}{1+z}$ about $z = 3$. Also state the region of validity. (7)

MODULE V

19. a) Expand $f(z) = \dfrac{1}{(z+1)(z+3)}$ as a Laurent series valid in the annulus $1 < |z| < 3$. (7)

b) Use Residue theorem to evaluate $\int_C \dfrac{z-3}{z^2+2z+5} dz$ where C is the circle $|z+1-i| = 2$. (7)

OR

20. a) Evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ where C is |z-2|=2 by using Cauchy Residue theorem. (7)

b) Using contour integration, evaluate $\int_0^\infty \frac{1}{(x^2+a^2)^2} dx$. (7)

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