Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

THIRD SEMESTER B.TECH DEGREE EXAMINATION (S), FEBRUARY 2023
COMPUTER SCIENCE AND ENGINEERING
(2020 SCHEME)

Course Code : **20MAT203**

Course Name: **Discrete Mathematical Structures**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Show that $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology.
2. Establish the validity of the following argument without using truth table
   $P \rightarrow q$
   $q \rightarrow r$
   $r \rightarrow s$
   ____
   $\therefore p \rightarrow s$
3. In how many ways can the letters of the word MATHEMATICS be arranged such that the vowels must always come together.
4. How many permutations of 1,2,3,4,5,6,7 are not derangements.
5. Explain POSET. Give example.
6. Let A= {1,2,3,4}, give an example of a relation which is Reflexive, Transitive and not Symmetric.
7. Determine the sequence generated by the exponential generating function $f(x)= e^x + x^2$
8. Determine the coefficient of x^{50} in $f(x) = (x^7 + x^8 + x^9 + \ldots)^6$.
9. Define a semi group. Give an example of a semi group which is not a monoid.
10. If (G, .) is a group prove that for all a, b ∈ G $(ab)^{-1} = b^{-1} a^{-1}$

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Check the validity of the following argument (7)
    Premises: $(p \rightarrow r)$, $(r \rightarrow s)$, $t \vee \neg s$, $\neg t \vee u$, $\neg u$
    Conclusion: $\neg p$

    b) Check the validity of the following argument without using truth table. (7)
    Premises: “If I join JNTU then I will get best education. If I get best education, then I will get job in USA. If I get job in USA then I will become a millionaire, I joined JNTU”.
    Conclusion: “I will become a millionaire”.

OR

12. a) Check the validity of the following argument using truth table (7)

(i). $[(p \to q) \land q] \to p$

(ii). $[(p \to q) \land (q \to r)] \to (p \to r)$

b) By the method of contradiction prove the validity of the following (7)

$p \to (q \land r)$

$r \to s$

$\neg(q \land s)$

$\therefore \neg p$

MODULE II

13. a) Find the coefficient of $w^3x^2yz^2$ in the expansion of $(2w-x+3y-2z)^8$ (6)

b) Determine the number of positive integers $1 \le n \le 10000$ where n is not divisible by 5, 6 and 8. (8)

OR

14. a) Find the number of arrangements in the word TALLAHASSEE. How many arrangements have no adjacent A's. (6)

b) Determine the number of integral solutions of $x_1 + x_2 + x_3 + x_4 = 32$ where (8)

a) $x_i \ge 0,\ 1 \le i \le 4$.

b) $x_i > 0,\ 1 \le i \le 4$

c) $x_1, x_2 \ge 5$ and $x_3, x_4 \ge 7$.

MODULE I7II

15. a) Let f, g and h be three functions from R to R defined by (6)

$f(x) = x^3 - 4x$, $g(x) = \frac{1}{x^2+1}$ and $h(x) = x^4$.

find (i). (f o g) o h (ii). f o (g o h) (iii) (h o g) o f

b) Let A={ 1,2,3,…,12} and R be a relation defined in A x A by (a, b) R (c, d) if and only if a+ d = b+ c. Prove that R is an equivalence relation. Also find the equivalence class of (2, 5). (8)

OR

16. a) Let Z be the set of all integers. R is a relation called "congruence modulo 3" defined by R={ (x,y) : x , y ∈ Z and x-y is divisible by 3} show that R is an equivalence relation? Determine the equivalence classes and partition of Z induced by R? (8)

b) Draw the Hasse Diagram of (D_{35}, /). Show that it is a lattice by using meet join table. (6)

MODULE IV

17. a) Solve the recurrence relation (6)
$a_n - 10a_{n-1} + 25a_{n-2} = 0,\ n \geq 2,\ a_0 = 0\ ,\ a_1 = 4$

b) Solve the recurrence relation (8)
$a_{n+2} - 8a_{n+1} + 16a_n = 8(5)^n,\ n \geq 0,\ a_0 = 12\ ,\ a_1 = 5$

OR

18. a) The number of bacteria in culture is 1000, and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. (6)

b) Solve the recurrence relation (8)
$a_{n+2} - 6a_{n+1} + 9a_n = 3(2)^n + 7(3)^n,\ n \geq 0,\ a_0 = 1,\ a_1 = 4$

MODULE V

19. a) Prove that the set $G=\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}, \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\right\}$ form a group under multiplication of matrices. (7)

b) Show that $< Z_6\ ,\ +_6 >$ is an abelian group where $+_6$ is the operation "addition modulo 6". (7)

OR

20. a) Let Q^+ denote the set of positive rational numbers. A composition " *"is defined in Q^+ by $a * b = \frac{ab}{2}$ for all a, b $\in Q^+$. Prove that $(Q^+, *)$ is a abelian group. (7)

b) If (G,.) is a group and a, b ∈ G, prove that $(ab)^2 = a^2b^2$ if and only if G is abelian.
(7)

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