Register No.: ………………………… Name: ……………………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER B.TECH DEGREE EXAMINATION (S), FEBRUARY 2023

(2020 SCHEME)

Course Code : **20MAT101**

Course Name: **LINEAR ALGEBRA AND CALCULAS**

Max. Marks : **100** **Duration: 3 Hours**

Non-Programmable Calculator May be Permitted

PART A

(Answer all questions. Each question carries 3 marks)

1. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$.
2. Find the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.
3. If $f(x, y, z) = x^3y^2z^4 + 2xy + z$, find $f_x(x, y, z),\ f_y(x, y, z)\ and\ f_z(x, y, z)$.
4. Find the rate of change of $z = \sin(y^2 - 4x)$ with respect to y at the point (3,1).
5. Evaluate the double integral $\int_0^1 \int_{-x}^{x^2} xy^2 dxdy$.
6. Evaluate the integral $\int_0^1 \int_0^1 \int_0^1 e^{(x+y+z)} dxdydz$.
7. Check whether the series $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$ converges or diverges.
8. Determine whether the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$ is convergent and if so, find its sum.
9. Find the Taylor series expansion of e^x about $x = -1$.
10. Obtain the Fourier coefficient a_0 of the function $f(x) = |x|$ in $-\pi < x < \pi$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Test for consistency and solve the following system of equations (7)

$$2x - y + 3z = 8$$
$$-x + 2y + z = 4$$
$$3x + y - 4z = 0$$

b) Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (7)

OR

12. a) For what values of λ *and* μ the given system of equations (7)

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

has (a) no solution (b) a unique solution and (c) infinite number of solutions.

b) Show that the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence, find P such that $P^{-1}AP$ is a diagonal matrix. (7)

MODULE II

13. a) Let f be differentiable function of 3 variables and suppose that $w = f(x - y, y - z, z - x)$. Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. (7)

b) Locate all the relative extrema and saddle points of $f(x, y) = x^2 + xy - 2y - 3x + 1$. (7)

OR

14. a) Given that $f(x, y) = y^3 e^{-5x}$, find (7)

(a) $f_{xyy}(0,1)$ (b) $f_{xxx}(0,1)$ (c) $f_{yyxx}(0,1)$

b) Given that $z = e^{xy},\ x = 2u + v,\ y = \frac{u}{v}$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain rule. (7)

MODULE III

15. a) Reverse the order of integration and hence evaluate $\int_0^1 \int_x^1 \frac{x}{x^2+y^2} dydx$. (7)

b) Find the area bounded by the x - axis, $y = 2x$ and x+$y = 1$ using the double integration. (7)

OR

16. a) Use a double integral to find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $z = 4 - 4x - 2y$. (7)

b) Find the mass of the lamina with density $\delta(x, y) = x + 2y$ is bounded by the x- axis, the line $x = 1$ and the curve $y = \sqrt{x}$. (7)

MODULE IV

17. a) Check whether the following series converges. (7)

i. $\sum_{k=1}^{\infty} \frac{1}{2k^2+k}$

ii. $\sum_{k=1}^{\infty} \frac{k^k}{k!}$

b) Discuss the convergence of the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}$ (7)

OR

18. a) Determine the rational number representing the decimal number 0.412412412… (7)

b) Check whether the following series converges. (7)

i. $\sum_{k=1}^{\infty} \frac{1}{(ln(k+1))^k}$

ii. $\sum_{k=1}^{\infty} k \left(\frac{1}{3}\right)^k$

MODULE V

19. a) Expand in a Fourier series, $f(x) = x$ in the interval $0 < x < 2\pi$. (7)

b) Find the Fourier cosine series expansion of $f(x) = x^2 - 2$ in $(0, 2)$. (7)

OR

20. a) Find the Fourier series to represent $x - x^2$ from $-\pi \; to \; \pi$ (7)

b) Obtain the half range sine series of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$ (7)

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