

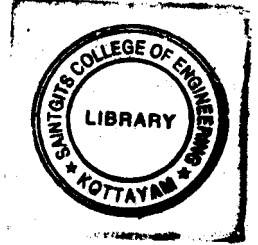
**B.TECH. DEGREE EXAMINATION, NOVEMBER 2014****Fifth Semester**

Branch : Common to all Branches except C.S. and I.T.

EN 010 501-A—ENGINEERING MATHEMATICS—IV

(Regular/Improvement/Supplementary)

[New Scheme—2010 Admission onwards]



Time : Three Hours

Maximum : 100 Marks

**Part A***Answer all questions.**Each question carries 3 marks.*

1. An electrostatic field in the  $xy$ -plane is given by the potential function  $\phi = 3x^2y - y^3$ , find the stream function.
2. Find the image of the circle  $|z - 1| = 1$  in the complex plane under the mapping  $w = \frac{1}{z}$ .
3. Find the real root of the equation  $x^2 - 2x - 5 = 0$  by the method of false position correct to 3 decimal places.
4. Solve  $\frac{dy}{dx} = 1 - y$ ,  $y(0) = 0$  in the range  $0 \leq x \leq 3$  by taking  $h = 0.1$  by the modified Euler's method.
5. Construct the dual of the L.P.P.  
 Maximize  $z = 4x_1 + 9x_2 + 2x_3$   
 subject to  $2x_1 + 3x_2 + 2x_3 \leq 7$ ,  $3x_1 - 2x_2 + 4x_3 = 5$ ;  $x_1, x_2, x_3 \geq 0$ .

(5 × 3 = 15 marks)

**Part B***Answer all questions.**Each question carries 5 marks.*

6. Show that  $\sqrt{|xy|}$  is not analytic at the origin, although Cauchy-Riemann equations are satisfied at the point.
7. Find the Taylor's series expansion of  $f(z) = \frac{2z^3 + 1}{z^2 + z}$  about  $z = i$ .

**Turn over**

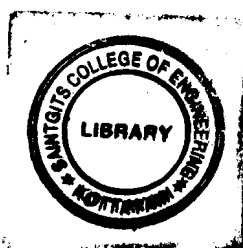
8. Find by the iteration method, a real root of  $2x - \log_{10}x = 7$ .
9. Solve  $\frac{dy}{dx} = x + z$ ,  $\frac{dz}{dx} = x - y^2$  with  $y(0) = 2$ ,  $z(0) = 1$  to get  $y(0.1)$ ,  $y(0.2)$ ,  $z(0.1)$  and  $z(0.2)$  approximately by Taylor's series.
10. Using graphical method, solve the following L.P.P.

$$\text{Maximize } z = 2x_1 + 3x_2$$

$$\text{subject to } x_1 - x_2 \leq 2$$

$$x_1 + x_2 \geq 4,$$

$$x_1, x_2 \geq 0.$$



(5 × 5 = 25 marks)

### Part C

Answer all questions.

Each full question carries 12 marks.

11. (a) Determine the analytic function  $f(z) = u + iv$  if  $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$  and  $f(\pi/2) = 0$ .  
(6 marks)
- (b) Find the bilinear transformation which maps the points  $z = 1, i, -1$  into the points  $w = i, 0, -i$ . Hence find the image of  $|z| < 1$ .  
(6 marks)

Or

12. (a) Prove that the function  $f(z)$  defined by  $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$ ,  $z \neq 0$  and  $f(0) = 0$  is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet  $f'(0)$  does not exist.  
(6 marks)
- (b) Show that the transformation  $w = \frac{3-z}{z-2}$  transforms the circle with center  $\left(\frac{5}{2}, 0\right)$  and radius  $\frac{1}{2}$  in the  $z$ -plane into the imaginary axis in the  $w$ -plane and the interior of the circle into the right half of the plane.  
(6 marks)

13. (a) Evaluate  $\int_C \frac{z-3}{z^2+2z+5} dz$ , where  $C$  is the circle (i)  $|z| = 1$ ; (ii)  $|z+1-i| = 2$ ; (iii)  $|z+1+i| = 2$ .  
(8 marks)

- (b) Determine the poles of the function  $f(z) = \frac{x^2}{(z-1)^2(z+2)}$  and the residue at each pole.  
(4 marks)

Or

14. (a) Find the Laurent's expansion of  $f(z) = \frac{7z-2}{(z+1)z(z-2)}$  in the region  $1 < |z+1| < 3$ . (5 marks)

(b) Show the method of residues, that  $\int_0^\pi \frac{a}{a^2 + \sin^2 \theta} d\theta = \frac{\pi}{\sqrt{1+a^2}}$ . (7 marks)

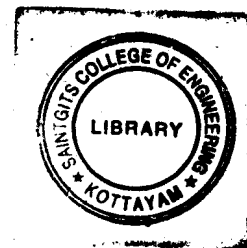
15. (a) Using Newton's iterative method, find the real root of  $x \log_{10} x = 1.2$  correct to five decimal places. (6 marks)

(b) Solve by Gauss-Seidel method :

$$10x + 2y + z = 9$$

$$2x + 20y - 2z = -44$$

$$-2x + 3y + 10z = 22.$$



(6 marks)

Or

16. (a) Find a real root of the equation  $x^3 - x - 11 = 0$ , correct to 4 decimal places using the bisection method. (6 marks)

(b) Find the root of the equation  $\cos x - xe^x = 0$  by secant method correct to four decimal places. (6 marks)

17. Using Runge-Kutta method of fourth order, solve  $\frac{dy}{dx} = yz + x$ ,  $\frac{dz}{dx} = xz + y$  given that  $y(0) = 1$ ,  $z(0) = -1$  for  $y(0.2)$ ,  $z(0.2)$ .

Or

18. Apply Milne's method, to find a solution of the differential equation  $y' = x - y^2$  in the range  $0 \leq x \leq 1$  for the boundary condition  $y = 0$  at  $x = 0$ .

19. (a) What is the maximization transport problem? How do you solve it? (3 marks)

(b) Using simplex method solve the LPP

$$\text{Maximize } z = 5x_1 + 3x_2$$

$$\text{subject to } x_1 + x_2 \leq 2$$

$$5x_1 + 2x_2 \leq 10$$

$$3x_1 + 8x_2 \leq 12,$$

$$x_1, x_2 \geq 0.$$

(9 marks)

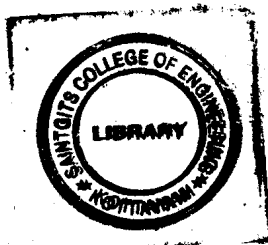
Or

Turn over

20. Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method (VAM). Here,  $F_1$ ,  $F_2$  and  $F_3$  are factories, and  $W_1$ ,  $W_2$  and  $W_3$  are warehouses.

	$W_1$	$W_2$	$W_3$	$W_4$	<i>Production of Factories</i>
$F_1$	21	16	25	13	11
$F_2$	17	18	14	23	13
$F_3$	32	27	18	41	19
Capacity of the warehouse	6	10	12	15	43

(5 × 12 = 60 marks)



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6. Show that  $\sqrt{|xy|}$  is not analytic at the origin, although Cauchy-Riemann equations are satisfied at the point.
7. Find the Taylor's series expansion of  $f(z) = \frac{2z^3 + 1}{z^2 + z}$  about  $z = i$ .

**Turn over**

8. Find by the iteration method, a real root of  $2x - \log_{10}x = 7$ .
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$$x_1 + x_2 \geq 4,$$

$$x_1, x_2 \geq 0.$$

(5 × 5 = 25 marks)



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Answer all questions.  
Each full question carries 12 marks.

11. (a) Determine the analytic function  $f(z) = u + iv$  if  $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$  and  $f(\pi/2) = 0$ . (6 marks)
- (b) Find the bilinear transformation which maps the points  $z = 1, i, -1$  into the points  $w = i, 0, -i$ . Hence find the image of  $|z| < 1$ . (6 marks)

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- (b) Determine the poles of the function  $f(z) = \frac{x^2}{(z-1)^2(z+2)}$  and the residue at each pole. (4 marks)

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14. (a) Find the Laurent's expansion of  $f(z) = \frac{7z-2}{(z+1)z(z-2)}$  in the region  $1 < |z+1| < 3$ . (5 marks)

(b) Show the method of residues, that  $\int_0^\pi \frac{a}{a^2 + \sin^2 \theta} d\theta = \frac{\pi}{\sqrt{1+a^2}}$ . (7 marks)

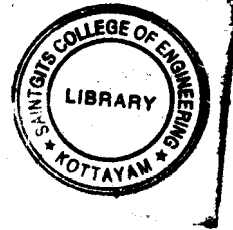
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Or

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17. Using Runge-Kutta method of fourth order, solve  $\frac{dy}{dx} = yz + x$ ,  $\frac{dz}{dx} = xz + y$  given that  $y(0) = 1$ ,  $z(0) = -1$  for  $y(0.2)$ ,  $z(0.2)$ .

Or

18. Apply Milne's method, to find a solution of the differential equation  $y' = x - y^2$  in the range  $0 \leq x \leq 1$  for the boundary condition  $y = 0$  at  $x = 0$ .

19. (a) What is the maximization transport problem? How do you solve it? (3 marks)

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$$5x_1 + 2x_2 \leq 10$$

$$3x_1 + 8x_2 \leq 12,$$

$$x_1, x_2 \geq 0.$$

(9 marks)

Or

Turn over

20. Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method (VAM). Here,  $F_1$ ,  $F_2$  and  $F_3$  are factories, and  $W_1$ ,  $W_2$  and  $W_3$  are warehouses.

	$W_1$	$W_2$	$W_3$	$W_4$	<i>Production of Factories</i>
$F_1$	21	16	25	13	11
$F_2$	17	18	14	23	18
$F_3$	32	27	18	41	19
Capacity of the warehouse	6	10	12	15	48

(5 × 12 = 60 marks)





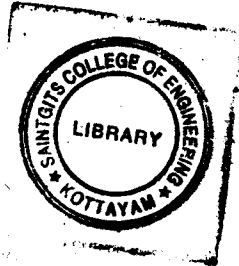
**B.TECH. DEGREE EXAMINATION, NOVEMBER 2014****Fifth Semester**

Branch : Computer Science and Engineering/Information Technology

EN 010 501-B—ENGINEERING MATHEMATICS—IV (CS, IT)

(Regular/Improvement/Supplementary)

[New Scheme—2010 Admission onwards]



Time : Three Hours

Maximum : 100 Marks

**Part A***Answer all questions.**Each question carries 3 marks.*

1. Evaluate  $\Delta (\sin 2x \cos 4x)$ .
2. Find the Z-transform of  $a^n \cos n\theta$ .
3. Let  $a$  be a numeric function such that  $a_r = \begin{cases} 2, & 0 \leq r \leq 3 \\ 2^{-r} + 5, & r \geq 4 \end{cases}$ . Determine  $\Delta a$  and  $\nabla a$ .
4. Evaluate  $\int_0^{1+i} (x^2 - iy) dz$  along the path  $y = x^2$ .
5. Explain the arrival pattern of customers.

(5 × 3 = 15 marks)

**Part B***Answer all questions.**Each question carries 5 marks.*

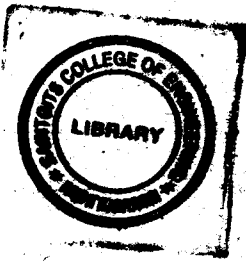
6. Using Newton's divided difference formula, Evaluate  $f(8)$  and  $f(15)$  given :

$x :$	4	5	7	10	11	13
$y :$	48	100	294	900	1210	2028

7. Use convolution theorem to find the inverse Z-transform of  $\frac{8z^2}{(2z-1)(4z+1)}$ .
8. Find the particular solution of the difference equation  $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$ .
9. If  $f(\xi) = \int_C \frac{3z^2 + 7z + 1}{z - \xi} dz$ , where  $C$  is the circle  $|z| = 2$ , find the values of  $f(3)$ ,  $f'(1-i)$  and  $f''(1-i)$ .
10. State and explain Little's formula. What are its applications ?

(5 × 5 = 25 marks)

**Turn over**



## Part C

Answer all questions.

Each full question carries 12 marks.

11. (a) Using Newton's forward interpolation formula, find
- $y$
- at
- $x = 8$
- from the following table :—

$x$ :	0	5	10	15	20	25
$y$ :	7	11	14	18	24	32

(6 marks)

- (b) Evaluate
- $\int_0^4 e^x dx$
- by Simpson's rule, given that
- $e = 2.72$
- ,
- $e^2 = 7.39$
- ,
- $e^3 = 20.09$
- ,
- $e^4 = 54.6$
- and compare it with the actual value.

(6 marks)

Or

12. (a) From the following table, calculate
- $\frac{dy}{dx}$
- ,
- $\frac{d^2y}{dx^2}$
- and
- $\frac{d^3y}{dx^3}$
- at
- $x = 1.5$

$x$ :	1.5	2.0	2.5	3.0	3.5	4.0
$y$ :	3.375	7.0	13.625	24.0	38.875	59.0

(6 marks)

- (b) Apply (i) Trapezoidal rule ; and (ii) Simpson's
- $\frac{1}{3}$
- rule, to find an approximate value of
- $\int_3^8 x^4 dx$
- by taking six equal subintervals. Compare it with the exact value.

(6 marks)

13. (a) Find
- $Z^{-1}\left(\frac{4z}{z-1^3}\right)$
- by the long division method. (6 marks)

- (b) Solve
- $y_{n+1} + 4y_{n+1} + 3y_n = 3^n$
- with
- $y_0 = 0$
- ,
- $y_1 = 1$
- . (6 marks)

Or

14. (a) Find the inverse Z-transform of
- $\frac{2(z^2 - 5z + 6.5)}{(z-2)(z-3)^2}$
- for
- $2 < |z| < 3$
- . (6 marks)

- (b) Solve
- $y_{n+2} - 2y_{n+1} + y_n = 3n + 5$
- . (6 marks)

15. (a) Determine the generating function of the numeric function
- $a_r$
- ,

$$\text{where } a_r = \begin{cases} 2^r, & \text{if } r \text{ is even} \\ 2^{-r}, & \text{if } r \text{ is odd} \end{cases}$$

(6 marks)

- (b) Solve the difference equation
- $a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r$
- . (6 marks)

Or

16. (a) Determine the discrete numeric function corresponding to the generating function  $A(z) = (1+z)^n + (1-z)^n$ . (6 marks)

(b) Determine the particular solution for the difference equation  $a_r - 3a_{r-1} + 2a_{r-2} = 2^r$ . (6 marks)

17. (a) Expand  $\frac{1}{z^2 - 3z + 2}$  in the region (i)  $|z| < 1$ ; (ii)  $1 < |z| < 2$ ; (iii)  $|z| > 2$ ; (iv)  $0 < |z - 1| < 1$ . (9 marks)

(b) Determine the poles of the function  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$  and the residue at each pole. (3 marks)

Or

18. (a) Evaluate by contour integration  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ . (9 marks)

(b) Expand the function  $\frac{1}{z+1}$ , about  $z = 1$  in Taylor's series. (3 marks)

19. (a) In a supermarket, the average arrival rate of customers is 10/hr. The average time taken at the bill and cash desk is 4.5 min. This time is exponentially distributed :

- (i) How long will be customer expect to wait for service at the cash desk ?
- (ii) What is the chance that the queue length will exceed 5 ?
- (iii) What is the probability that the cashier is working ?

(7 marks)

(b) Explain the different service disciplines in the case of a queuing system. (5 marks)

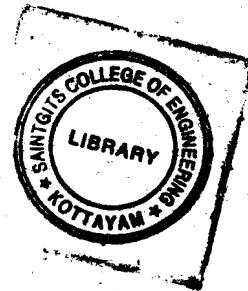
Or

20. (a) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the servicetime distribution is also exponential with an average 36 minutes. Calculate the following :—

- (i) The mean queue size.
- (ii) The probability that the queue size exceeds 10.
- (iii) If the input of trains increases to an average 33 per day, what will be the change in (i) and (ii).

(7 marks)

Turn over

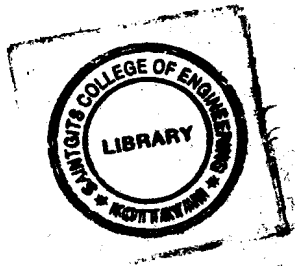


(b) A T.V. repairmen finds that the time spent on his job has an exponential/distribution with mean 30 minutes. If the repaired set arrive on an average of 10 per 8-hour day with Poisson :

- (i) What is the repairmen's idle time each day ?
- (ii) What is the average queue length ?
- (iii) Find average number of jobs in the system.

(5 marks)

[5 × 12 = 60 marks]



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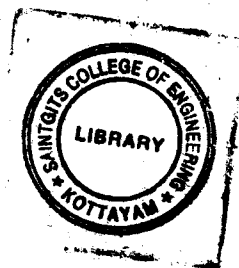
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**Part B***Answer all questions.**Each question carries 5 marks.*

6. Using Newton's divided difference formula, Evaluate  $f(8)$  and  $f(15)$  given :
 

$x:$	4	5	7	10	11	13
$y:$	48	100	294	900	1210	2028
7. Use convolution theorem to find the inverse Z-transform of  $\frac{8z^2}{(2z-1)(4z+1)}$ .
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Answer all questions.

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$x$ :	0	5	10	15	20	25
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- (b) Evaluate  $\int_0^4 e^x dx$  by Simpson's rule, given that  $e = 2.72$ ,  $e^2 = 7.39$ ,  $e^3 = 20.09$ ,  $e^4 = 54.6$  and compare it with the actual value.

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12. (a) From the following table, calculate  $\frac{dy}{dx}$ ,  $\frac{d^2y}{dx^2}$  and  $\frac{d^3y}{dx^3}$  at  $x = 1.5$

$x$ :	1.5	2.0	2.5	3.0	3.5	4.0
$y$ :	3.375	7.0	13.625	24.0	38.875	59.0

(6 marks)

- (b) Apply (i) Trapezoidal rule ; and (ii) Simpson's  $\frac{1}{3}$  rule, to find an approximate value of  $\int_3^9 x^4 dx$  by taking six equal subintervals. Compare it with the exact value.

(6 marks)

13. (a) Find  $Z^{-1}\left(\frac{4z}{z-1^8}\right)$ , by the long division method. (6 marks)

- (b) Solve  $y_{n+1} + 4y_{n+1} + 3y_n = 3^n$  with  $y_0 = 0, y_1 = 1$ . (6 marks)

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14. (a) Find the inverse Z-transform of  $\frac{2(z^2 - 5z + 6.5)}{(z-2)(z-3)^2}$  for  $2 < |z| < 3$ . (6 marks)

- (b) Solve  $y_{n+2} - 2y_{n+1} + y_n = 3n + 5$ . (6 marks)

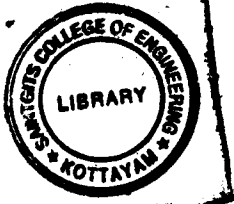
15. (a) Determine the generating function of the numeric function  $a_r$ ,

$$\text{where } a_r = \begin{cases} 2^r, & \text{if } r \text{ is even} \\ 2^{-r}, & \text{if } r \text{ is odd} \end{cases}$$

(6 marks)

- (b) Solve the difference equation  $a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r$ . (6 marks)

Or



16. (a) Determine the discrete numeric function corresponding to the generating function  $A(z) = (1+z)^n + (1-z)^n$ .

(6 marks)

(b) Determine the particular solution for the difference equation  $a_r - 3a_{r-1} + 2a_{r-2} = 2^r$ .

(6 marks)

17. (a) Expand  $\frac{1}{z^2 - 3z + 2}$  in the region (i)  $|z| < 1$ ; (ii)  $1 < |z| < 2$ ; (iii)  $|z| > 2$ ; (iv)  $0 < |z - 1| < 1$ .

(9 marks)

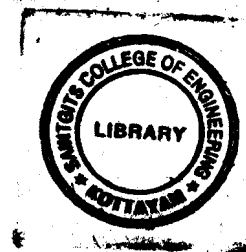
(b) Determine the poles of the function  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$  and the residue at each pole.

(3 marks)

Or

18. (a) Evaluate by contour integration  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$

(9 marks)



(b) Expand the function  $\frac{1}{z+1}$ , about  $z = 1$  in Taylor's series.

(3 marks)

19. (a) In a supermarket, the average arrival rate of customers is 10/hr. The average time taken at the bill and cash desk is 4.5 min. This time is exponentially distributed:

- (i) How long will be customer expect to wait for service at the cash desk?
- (ii) What is the chance that the queue length will exceed 5?
- (iii) What is the probability that the cashier is working?

(7 marks)

(b) Explain the different service disciplines in the case of a queuing system.

(5 marks)

Or

20. (a) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the servicetime distribution is also exponential with an average 36 minutes. Calculate the following:—

- (i) The mean queue size.
- (ii) The probability that the queue size exceeds 10.
- (iii) If the input of trains increases to an average 33 per day, what will be the change in (i) and (ii).

(7 marks)

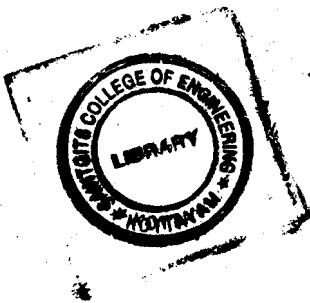
Turn over

(b) A T.V. repairmen finds that the time spent on his job has an exponential/distribution with mean 30 minutes. If the repaired set arrive on an average of 10 per 8-hour day with Poisson :

- (i) What is the repairmen's idle time each day ?
- (ii) What is the average queue length ?
- (iii) Find average number of jobs in the system.

(5 marks)

[5 × 12 = 60 marks]





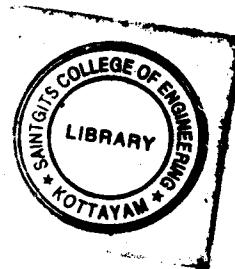
**B.TECH. DEGREE EXAMINATION, NOVEMBER 2014****Fifth Semester**

Branch : Common to all Branches except C.S. and I.T.

**ENGINEERING MATHEMATICS - IV (CMELPASUF)**

(Old Scheme—Supplementary/Mercy Chance)

[Prior to 2010 admissions]



Time : Three Hours

Maximum : 100 Marks

*Answer all questions.**Each full question carries 20 marks.*

1. (a) Using Cauchy's integral formula, evaluate  $\int_C \frac{z+1}{z^2+2z+5} dz$  where C is the circle  $|z+1-i|=2$ , integration being taken in the counter clockwise direction.

- (b) Expand  $\frac{1}{z(z-1)(z-2)}$  in Laurent's series for  $|z| > 2$ .

Or

2. (a) Evaluate  $\oint_C \frac{z}{z(z-1)(z-2)^2} dz$ , where C is the circle  $|z-2| = \frac{1}{2}$ .

- (b) Evaluate by contour integration  $\int_0^{\infty} \frac{x^2 dx}{(x^2+9)(x^2+4)^2}$ .

3. (a) Find a root of the equation  $x^6 - x^4 - x^3 = 1$  correct to three decimal places using Regula Falsi method.
- (b) Solve by Gauss-Jacobi's method :

$$54x + y + z = 110$$

$$2x + 15y + 6z = 72$$

$$-x + 6y + 27z = 85.$$

Or

Turn over

4. (a) Find a root of  $x^3 - 4x - 9 = 0$  correct to three decimal places using Bisection method.  
 (b) Solve by Gauss-Seidel method :

$$10x_1 - 5x_2 - 2x_3 = 3$$

$$4x_1 - 10x_2 + 3x_3 = -3$$

$$x_1 + 6x_2 + 10x_3 = -3.$$

5. (a) Using Taylor's series method solve  $\frac{dy}{dx} = x^2 - y, y(0) = 1$  at  $x = 0.1, 0.2, 0.3$  and  $0.4$ .

- (b) Use Runge-Kutta method to solve  $\frac{dy}{dx} = x^2 - y^2, y(1) = 1.5$  at  $x = 1.2$  in steps of  $0.1$ .

Or

6. (a) Taking  $h = 0.05$  and applying modified Euler's method, solve the initial value problem  $y' = x^2 + y, y(0) = 1$ , obtain  $y(0.1)$ .

- (b) Using Milne's predictor-corrector method solve the initial value problem

$$\frac{2dy}{dx} = (1 + x^2)y^2, y(0) = 1 \text{ and obtain } y(0.4). \text{ Use the solution values :}$$

$$y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21.$$

7. (a) Given  $Z(u_n) = \frac{2z^2 + 3z + 4}{(z-3)^3}, |z| > 3$ , show that  $u_1 = 2, u_2 = 21, u_3 = 139$ .

- (b) Solve  $x_{n+1} - y_n = 1, y_{n+1} - x_n = 0, x_0 = 0, y_0 = -1$ .

Or

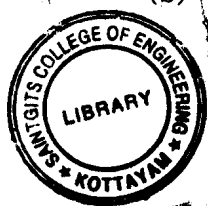
8. (a) Find the  $z$ -transform of :

(i)  $e^{4t} \sin 3t$ .

(ii)  $(t + T)e^{-(t+T)}$ .

(iii)  $4^n + \left(\frac{1}{2}\right)^n + u(n-3)$ .

- (b) Find the inverse  $z$ -transform of  $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$ .



9. (a) Solve the following L.P.P. by simplex method :

$$\text{Maximize } Z = x_1 + 2x_2 + 3x_3 - x_4$$

$$\text{subject to } x_1 + 2x_2 + 3x_3 = 15$$

$$2x_1 + x_2 + 5x_3 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 = 10.$$

- (b) Solve the following transportation problem :

		Destination				Avail
		D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
Origin	O <sub>1</sub>	5	3	6	2	19
	O <sub>2</sub>	4	7	9	1	37
	O <sub>3</sub>	3	4	7	5	34
Require		16	18	31	25	

Or

10. (a) Using the duality theory, solve the L.P.P. :

$$\text{Minimize } Z = 3x_1 - 2x_2 + 4x_3$$

$$\text{subject to } 3x_1 + 5x_2 + 4x_3 \geq 7$$

$$6x_1 + x_2 + 3x_3 \geq 4$$

$$7x_1 - 2x_2 - x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0.$$

- (b) Apply Vogel's method to find the transportation cost to the following transportation model :

	1	2	3	4	
1	10	2	20	11	15
2	12	7	9	20	25
3	4	14	16	18	10
	5	15	15	15	

(5 × 20 = 100 marks)

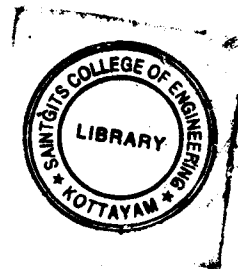
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