Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

THIRD SEMESTER B.TECH DEGREE EXAMINATION (Regular), DECEMBER 2022
COMPUTER SCIENCE AND ENGINEERING
(2020 SCHEME)

Course Code : **20MAT203**

Course Name: **Discrete Mathematical Structures**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Use truth table to verify the following logical equivalences:

   $P \rightarrow (q \wedge r) \Leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$

2. Define contradiction with example.
3. Show that if 8 people are in a room, at least two of them have birthdays that occur in the same day of the week.
4. In how many ways can the letters of the word 'ARRANGE' be arranged such that the two R's do not occur together.
5. Let f: $R \rightarrow R$ and g: $R \rightarrow R$ defined by $f(x) = x^2$, $g(x) = x + 5$. Show that $(f \circ g) \neq (g \circ f)$.
6. Let A ={1,2,3,4}, give an example of a relation which is Reflexive, Symmetric and not Transitive.
7. Determine the sequence generated by the generating function $f(x) = \frac{x^3}{1-x^2}$
8. Determine the coefficient of x^{15} in $f(x) = (x^2 + x^3 + x^4 + \ldots)^4$.
9. Define a monoid. Give an example.
10. Prove that every cyclic group is abelian.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Check the validity of the following argument without using truth table
"Rita is baking a cake, If Rita is baking a cake then she is not practicing her flute. If Rita is not practicing her flute, then her father will not buy her a car. Therefore, Rita's father will not buy her a car." (7)

b) (7)

Check the validity of the following argument

(i). $[(p \rightarrow q) \land q] \rightarrow p$

(ii). $[(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)$

OR

12. a) Check the validity of the following argument (7)

Premises: $(p \rightarrow r)$, $(r \rightarrow s)$, $t \lor \neg s$, $\neg t \lor u$, $\neg u$

Conclusion: $\neg p$

b) By the method of contradiction prove the validity of the following (7)

$p \rightarrow (q \land r)$

$r \rightarrow s$

$\neg(q \land s)$

$\therefore \neg p$

MODULE II

13. a) How many arrangements are there of all the letters in the word SOCIOLOGICAL. (7)

(i) How many of these arrangements are A and G adjacent?

(ii) How many of these arrangements are all the vowels adjacent?

b) Determine the number of positive integers $1 \leq n \leq 10000$ where n is not divisible by 5, 6 and 8. (7)

OR

14. a) Find the coefficient of $w^3x^2yz^2$ in the expansion of $(2w-x+3y-2z)^8$ (6)

b) Determine the number of integral solutions of $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$ where $x_i \geq 0$, $1 \leq i \leq 6$. How many such solutions are there in the inequality $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 10$. (8)

MODULE III

15. a) Let A={1,2,3} , B={ 2,5} and C={2,4} Prove that (6)

(i). $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii). $A \times (B \cup C) = (A \times B) \cup (A \times C)$

b) Let A= {1,2,3,…,12} and R be a relation defined in A x A by (a, b) R (c, d) if and only if a+ d = b+ c. Prove that R is an equivalence relation. Also find the equivalence class of (2, 3). (8)

OR

16. a) Define a Poset. Let S be any set and P(S) be the power set of S. Prove that (P(S), ⊆) is a Poset. (8)

b) Draw the Hasse Diagram of (D_{42}, /). Find the compliment of each element in D_{42} (6)

MODULE IV

17. a) Solve the recurrence relation (6)
$a_n + a_{n-1} - 6a_{n-2} = 0$, $n \geq 2$, $a_0 = -1$, $a_1 = 8$

b) Solve the recurrence relation (8)
$a_n - 3a_{n-1} = 5(7)^n$, $n \geq 1$, $a_0 = 2$.

OR

18. a) A bank pays 6% annual interest on savings compounding the interest monthly. If Roy deposits Rs. 2000 on the first day of April, find using recurrence relation, how much will this deposit be worth a year later. (6)

b) Solve the recurrence relation (8)
$a_{n+2} + 3a_{n+1} + 2a_n = 3^n$, $n \geq 0$, $a_0 = 0$, $a_1 = 1$

MODULE V

19. a) If (G,.) is a group and a, b ∈ G, prove that $(ab)^2 = a^2b^2$ if and only if G is abelian. (6)

b) Show that < Z_7^*, . > is an abelian group where " . " is the operator “multiplication modulo 7". (8)

OR

20. a) Let S={1,2,3}. List all elements of S_3. Show that S_3 form a non-abelian group with respect to composition of permutations. (6)

b) Show that Q^+ of all positive rational numbers form an abelian group under the operation * defined by $a*b = \frac{ab}{2}$ where a, b ∈ Q^+. (8)

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