

Register No.: ..... Name: .....

**SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)**

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

**FIRST SEMESTER M.TECH DEGREE EXAMINATION (Regular), DECEMBER 2022****COMPUTER SCIENCE AND SYSTEMS ENGINEERING****(2021 Scheme)****Course Code: 21SE101****Course Name: Discrete Structures for Computer Science****Max. Marks: 60****Duration: 3 Hours****PART A****(Answer all questions. Each question carries 3 marks)**

1. Let  $f, g$  and  $h$  be three functions from  $\mathbb{R}$  to  $\mathbb{R}$  defined by  $f(x) = x^3 - 4x$ ,  
 $g(x) = \frac{1}{x^2+1}$  and  $h(x) = x^4$ . find (i)  $(f \circ g) \circ h$  (ii)  $f \circ (g \circ h)$
2. Define lattice. Give example.
3. Show that  $p \rightarrow (p \vee \neg q)$  is a tautology
4. In how many ways can the letters of the word 'ARRANGE' be arranged such that the two R's do not occur together.
5. Define group with example.
6. If  $G$  is a group prove that  $(a^{-1})^{-1} = a, \forall a \in G$
7. Define unit with an example
8. Find the multiplicative inverse of 100 in  $Z_{1009}$

**PART B****(Answer one full question from each module, each question carries 6 marks)****MODULE I**

9. Let  $A = \{1, 2, 3, 4, 5\}$  and  $R$  be the relation defined on  $A \times A$  defined by (6)  
 $R = \{(a, b)R(c, d) ; a + b = c + d \text{ where } a, b, c, d \in A\}$ . Verify that  $R$  is an equivalence relation on  $A$ . Determine the equivalence classes of  $(1, 3)$ .

**OR**

10. Let  $Z$  be the set of all integers,  $R$  be the relation congruence modulo 5 (6)  
defined by  $R = \{(x, y) / x, y \in Z \ \& \ x - y \text{ is divisible by } 3\}$ . Show that  $R$  is an Equivalence relation. Determine the equivalence class and partition of  $Z$  induced by  $R$ .

**MODULE II**

11. Define a Poset. Show that  $\leq$  is a partial ordering on the set of all integers. (6)  
Draw the Hasse diagram  $(\mathbb{Z}, \leq)$ .

**OR**

12. Define Complemented lattice. Check whether  $(D_{42}, /)$  is a Complemented lattice or not. (6)

**MODULE III**

13. Show that the following argument is valid without using truth table. (6)  
"It is not sunny in the afternoon and it is colder than yesterday. We will go for swimming if it sunny. If we do not go for swimming, we will take a trip. If we take a trip, then we will be home by sunset. Therefore we will be at home".

**OR**

14. Show that  $\neg(p \vee q) \equiv \neg p \wedge \neg q$  (6)

**MODULE IV**

15. Determine the number of integral solutions of  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$  where  $x_i \geq 0$ ,  $1 \leq i \leq 6$ . How many such solutions are there in the inequality  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 10$ . (6)

**OR**

16. Let  $X$  be the binomial random variable that consists the number of success, each with probability  $p$ , among  $n$ , Bernoulli trials. Prove that  $E(x) = np$ . (6)

**MODULE V**

17. State and prove Lagrange's Theorem on Groups. (6)

**OR**

18. Prove that every group of prime order is cyclic. (6)

**MODULE VI**

19. Is 25 a unit in  $Z_{72}$ . If so find the multiplicative inverse of 25. (6)

**OR**

20. Determine whether  $\langle \mathbb{Z}, \oplus, \odot \rangle$  is a ring with binary operation  $x \oplus y = x + y - 7$  (6)  
and  $x \odot y = x + y - 3xy$  for every  $x, y$  in  $\mathbb{Z}$ .

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