Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH DEGREE EXAMINATION (Regular), DECEMBER 2022

ROBOTICS AND AUTOMATION

(2021 Scheme)

Course Code: **21RA101**

Course Name: **Advanced Mathematics and Optimization Techniques**

Max. Marks: **60** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the coordinate representation of the vector $[1 \quad 2 \quad 3]$ in R^3 with respect to the basis $S = \{[1 \quad 1 \quad 0], [1 \quad 0 \quad 1], [0 \quad 1 \quad 1]\}$ considered as row vectors.

2. Determine whether the transformation T is linear if $T: R^2 \longrightarrow R^1$ is defined by $T[a \quad b] = ab$ for all real numbers a and b.

3. Find $\langle x, y \rangle$ for $x = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{8} \end{bmatrix}$ and $y = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$ and determine whether x and y are orthogonal.

4. Define (i) decision variable (ii) objective function (iii) constraints.

5. What are the steps to formulate a linear programming problem?

6. A company produce two types of hats. Each hat of the first type requires twice as much labor time as the second type. If all the hats are of the second type only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and the second type to 150 and 250 hats. Assuming that the profit per hat is Rs.8 for type 1 and Rs.5 for type 2. Formulate the problem as a LP model in order to determine the number of hats to be produced for each type so as to maximize the profit.

7. Solve graphically
   $Minimize\ z = 3x_1 + 4x_2$
   $Subject\ to\ x_1 + 2x_2 \geq 4$
   $3x_1 + 2x_2 \geq 6$
   $x_1, x_2 \geq 0.$

8. State Kuhn-Tucker conditions for a nonlinear programming problem having a maximization objective function.

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. Determine whether the set P^n containing the identically zero polynomial and all polynomials of degree n or less in the variable t with real coefficients is a vector space under standard addition and scalar multiplication for polynomials, if the scalars are restricted to being real. (6)

OR

10. Prove that $S = \{[1 \quad 1 \quad 0],\ [0 \quad 1 \quad 1],\ [1 \quad 3 \quad 1]\}$ is a basis for R^3 considered as row vectors. (6)

MODULE II

11. Find the matrix representation for $T: R^3 \longrightarrow R^2$ defined by $T\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2a+3b-c \\ 4b+5c \end{bmatrix}$ with respect to the given bases $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ for R^3 and $C = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ for R^2. (6)

OR

12. Find the transition matrices between the two bases $G = \{t+1,\ t-1\}$ and $H = \{2t+1\ ,\ 3t+1\}$ for P^1 and verify $P_H^G v_H = v_G$ for the coordinate representations of the polynomial $3t+5$ with respect to each basis. (6)

MODULE III

13. Construct a QR decomposition for $\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 3 & 5 \end{bmatrix}$. (6)

OR

14. Find the least-squares solution to (6)

$$x + 2y + z = 1$$
$$3x - y = 2$$
$$2x + y - z = 2$$
$$x + 2y + 2z = 1$$

MODULE IV

15. Solve by Big M method (6)
$Minimize\ z = 9x_1 + 10x_2$
$Subject\ to\ x_1 + 2x_2 \geq 25$
$4x_1 + 3x_2 \geq 24$
$3x_1 + 2x_2 \geq 60$
$x_1, x_2 \geq 0.$

OR

16. Solve by Dual Simplex method (6)
$Maximize\ z = -4x_1 - 3x_2$
$Subject\ to\ x_1 + x_2 \leq 1$
$x_2 \geq 1$
$-x_1 + 2x_2 \leq 1$
$x_1, x_2 \geq 0.$

MODULE V

17. Solve by Gomory's cutting plane method (6)
$Maximize\ z = 4x_1 + 6x_2 + 2x_3$
$Subject\ to\ 4x_1 - 4x_2 \leq 5$
$-x_1 + 6x_2 \leq 5$
$-x_1 + x_2 + x_3 \leq 5$
$x_1, x_2, x_3 \geq 0$
$x_1, x_3\ are\ integers.$

OR

18. Solve the following integer programming problem using branch and bound method. (6)
$Maximize\ z = x_1 + 4x_2$
$subject\ to\ 2x_1 + 4x_2 \leq 7$
$5x_1 + 3x_2 \leq 15$
$x_1, x_2 \geq 0\ and\ are\ integers.$

MODULE VI

19. Solve the non linear programming problem using Lagrangian method (6)
Maximize $z = 4x_1 - 0.02x_1^2 + x_2 - 0.02x_2^2$
$Subject\ to\ x_1 + 2x_2 = 120$
$x_1, x_2 \geq 0$

OR

20. Solve the following using Kuhn-Tucker conditions (6)
$Maximise\ z = 3x_1^2 + 14x_1x_2 - 8x_2^2$
$Subject\ to\ 3x_1 + 6x_2 \leq 72$
$x_1, x_2 \geq 0$

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