Register No.: ……………………………… Name: ……………………………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (S), SEPT 2022

COMPUTER SCIENCE AND ENGINEERING

(2020 SCHEME)

Course Code: **20MAT206**

Course Name: **Graph Theory**

Max. Marks: **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Define a bipartite graph with an example. What is the number of vertices and edges of the complete bipartite graph $K_{101,102}$?
2. Is it possible to have a group of 9 people, each knowing exactly 7 others? Justify.
3. Define Euler graph. Give an example of a graph which is Euler as well as Hamiltonian.
4. Distinguish between reflexive digraph and transitive digraph.
5. Define minimally connected graph. Prove that a tree is minimally connected
6. Explain metric with an example.
7. Define cut set. Prove that for a tree every edge is a cut set.
8. Prove that K_5 is non planar.
9. Define incidence matrix of a graph.
10. Distinguish between Maximal matching and Perfect matching.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Define complete graph with an example. Obtain the number of edges of a complete graph with n vertices. (7)

    b) Define degree of a vertex. Show that for any graph the number of vertices of odd degree is always even. (7)

OR

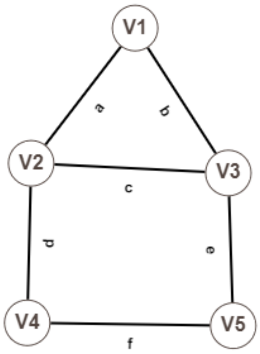
12. a) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges (7)

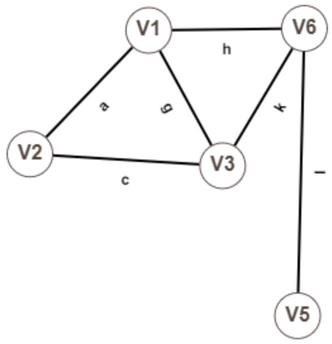
    b) Explain the terms walk, path and cycles with examples. (7)

MODULE II

13. a) Prove that in a complete graph K_n, $n \geq 3$ *is odd*, there are $\frac{(n-1)}{2}$ edge disjoint Hamiltonian cycles. (7)

b) Find the union, intersection and ring sum of the graphs G_1 and G_2 (7)





OR

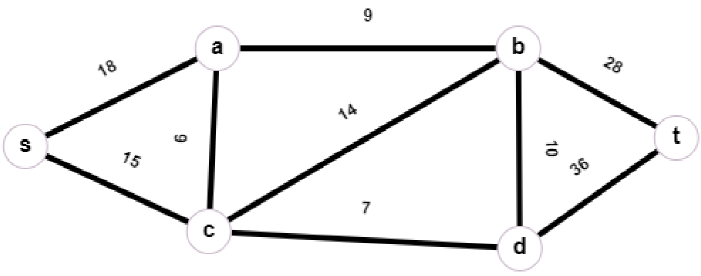
14. a) Explain travelling salesman problem. How it is related to Hamiltonian circuits. (7)

b) Prove that a Euler graph can be decomposed into edge disjoint cycles. (7)

MODULE III

15. a) Define spanning tree. Prove that every connected graph has at least one spanning tree (7)

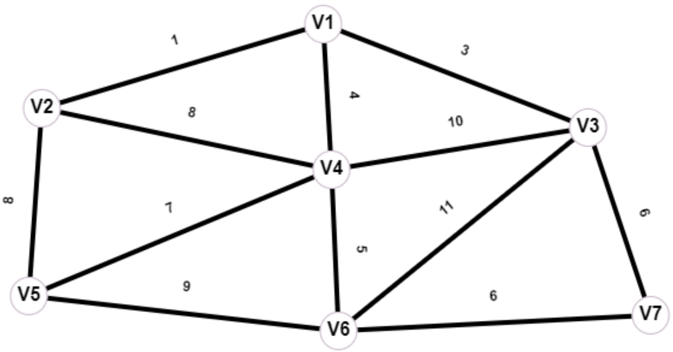
b) Write Dijkstra's algorithm. Using it find the length of the shortest path from s to t (7)



OR

16. a) Prove that every tree has one or two centers. (7)

b) Write Prims algorithm find the minimal spanning tree of the following graph. (7)



MODULE IV

17. a) Prove that a graph is k −connected if and only if there exist at least k-disjoint paths between any pair of vertices in G (7)

b) Define Geometric dual of a graph. What is the relationship between planar graph and its dual? (7)

OR

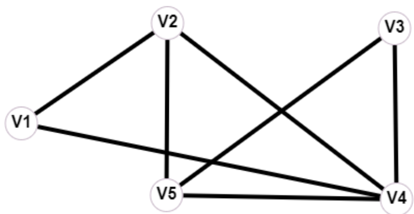
18. a) Define cut vertex. Prove that every internal vertex of a tree is a cut vertex. (7)

b) Prove that a connected planar graph with n vertices and e edges has $e - n + 2$ faces (7)

MODULE V

19. a) Define chromatic number. Prove that a non-empty graph is $2-$ chromatic if and only if it is bipartite. (8)

b) Define adjacency matrix. Find the adjacency matrix of the following graph.

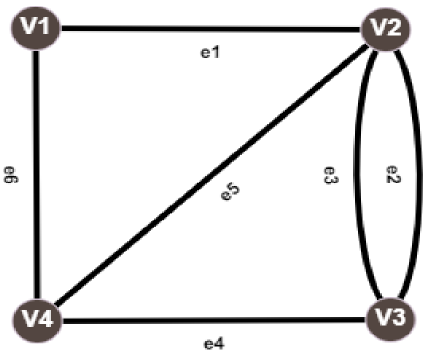


(6)

OR

20. a) Prove that every planar graph is $5-$ colourable. (8)

b) List the cycles and obtain the cycle matrix of the following graph. (6)


V1
V2
V3
V4
e1
e2
e3
e4
e5
e6

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