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# SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

**FOURTH SEMESTERB.TECH DEGREE EXAMINATION (S), SEPT 2022**

**COMMON TO EE, EC**

**(2020 SCHEME)**

**Course Code:** **20MAT204**

**Course Name:** **Probability, Random Processes and Numerical Methods**

**Max. Marks:** **100** **Duration: 3 Hours**

*(Non-programmable scientific calculators and statistical tables may be permitted)*

## PART A

*(Answer all questions. Each question carries 3 marks)*

1. $X$ follows a Poisson distribution such that $P[X=2]=P[X=3]$.Find the mean and $P[X=4]$
2. Determine the binomial distribution for which mean is 2 and variance is $\frac{4}{3}$
3. If $X$ follows an exponential distribution with $P[X \leq 1]=P[X>1]$ . Find the mean and variance of $X$
4. Random variable $X$ is uniformly distributed in the interval $(-k, k)$.Find the value of $k$ if $P[X \geq 1]=\frac{1}{3}$
5. Define wide sense stationary random process
6. Compute the variance of the random process $X(t)$ whose autocorrelation function is given by $R_{XX}(\zeta)=25+\frac{4}{1+6\zeta^2}$
7. Use Trapezoidal rule to evaluate $\int_0^1 e^{-x^2/2}\, dx$ considering 5 subintervals.
8. Find a root between 1 and 2 for $sinx=\frac{x}{2}$ using Newton-Raphson method, correct to three decimal places.
9. Use Runge-Kutta method of second order to find $y(0.1)$ for $\frac{dy}{dx}=y+\sin x, y(0)=2$ (Take $h=0.1$).
10. Given $\frac{dy}{dx}=\frac{y^2-2x}{y}, y(0)=1$.Use Euler's method with $h=0.1$ to compute the value of $y(0.2)$.

## PART B

*(Answer one full question from each module, each question carries 14marks)*

### MODULE I

11. a) A Random variable $X$ has the following probability distribution (7)

| x | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| f(x) | $\frac{1}{10}$ | $15k^2$ | $\frac{1}{5}$ | $2k$ | $\frac{3}{10}$ | $3k$ |

Compute the following

i. $k$

ii. $P[X < 2]$

iii. $P[-2 < X < 2]$

iv. $P[X \leq 2 \mid X > 0]$

v. Mean.

b) If on an average 9 ships out of 10 return safely to a port, what is the probability that out of 5 ships, 3 will arrive safely to the port? (7)

**OR**

12. a) Let $X$ and $Y$ be two random variables with joint pmf $p(x,y) = \frac{x+2y}{18}, x = 1,2$ and $y = 1,2$. Find the marginal pmf's of $X$ and $Y$. Are $X$ and $Y$ independent? (7)

b) Prove that a binomial distribution can be approximated to Poisson distribution when $n$ is large, $p$ is small and $np = \lambda$ (7)

**MODULE II**

13. a) Given a distribution with unknown mean $\mu$ and variance 1.5.Use Central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be atleast 0.95 that the sample mean will be within 0.5 of the population mean (7)

b) Derive the formula for mean and variance of uniform distribution (7)

**OR**

14. a) If $f(x,y) = 2 - x - y$ for $0 \leq x \leq 1, 0 \leq y \leq 1$ is the joint pdf of $(X,Y)$.Test whether $X$ and $Y$ are independent? (7)

b) $X$ is a normal random variable with mean 50 and standard deviation 10.Find $A$ and $B$ such that $P[X < A] = 0.10$ and $P[X > B] = 0.05$ (7)

**MODULE III**

15. a) Let $X(t) = Bcos(50t + \theta)$, where $B$ and $\theta$ are independent random variables. $B$ is a random variable with mean 0 and variance 1. $\theta$ is uniformly distributed in the interval $[-\pi, \pi]$.Find the mean and autocorrelation of the process. (7)

b) Prove that inter arrival time of a Poisson process with parameter $\lambda$ has an exponential distribution with mean $1/\lambda$. (7)

**OR**

16. a) Show that the random process $X(t) = Acos(\omega t + \theta)$ is WSS if $A$ and $\omega$ are constants and $\theta$ is a uniformly distributed random variable in $(0,2\pi)$ (7)

b) Find the autocorrelation and average power of the random process with power spectral density $S_{XX}(\omega) = \begin{cases} \alpha & -B \leq \omega \leq B \\ 0 & otherwise \end{cases}$. (7)

**MODULE IV**

17. a) Using Lagrange's Interpolation method, find the polynomial $f(x)$ to the data (7)

$f(1) = 1, f(3) = 27, f(4) = 64$ and hence find $f(2)$.

b) Find Newton's backward difference form of interpolating polynomial for the data
$f(4) = 19, f(6) = 40, f(8) = 79, f(10) = 142$.
Hence interpolate $f(9)$. (7)

**OR**

18. a) Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ with $n = 6$ by
(a)Trapezoidal rule
(b)Simpson's 1/3 rule (7)

b) In the given data the values of y are consecutive terms of a series of which 23.6 is the sixth term. Find the first and tenth terms of the series (7)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| y | 4.8 | 8.4 | 14.5 | 23.6 | 36.2 | 52.8 | 73.9 |

**MODULE V**

19. a) Evaluate $y(0.1)$ using Runge-Kutta Fourth order method for the differential equation $\frac{dy}{dx} = e^x + y$ and $y(0) = 0$. (Take h = 0.1) (7)

b) Use the method of least squares to fit an equation of the form y = ax + b to the following data (7)
y(1) = 6, y(2) = 7, y(3) = 9, y(4) = 10, y(5) = 12

**OR**

20. Use Gauss-Seidel method to solve the following system of equations (14)

$$5x - y = 9$$
$$-x + 5y - z = 4$$
$$-y + 5z = -6$$

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