Register No.: ………………………… Name: ………………………………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)
SECOND SEMESTER B.TECH DEGREE EXAMINATION (S), SEPT 2022
(2020 SCHEME)

Course Code :   **20MAT102**

Course Name: **Vector Calculus, Differential Equations and Transforms**

Max. Marks :   **100**   **Duration: 3 Hours**

PART A
(Answer all questions. Each question carries 3 marks)

1. Find the directional derivative of $f(x,y) = xe^y$ at (1,1) in the direction of – ve y axis
2. Prove that $\bar{F} = (x + 3y)\,\hat{\imath} + (y - 3z)\,\hat{\jmath} + (x - 2z)\,\hat{k}$ is solenoidal
3. Evaluate by Green's theorem $\int_c ydx + xdy$ where C is the unit circle.
4. Evaluate $\iint_\sigma \bar{F} . \hat{n}\, dS$ where $\bar{F} = ax\,\hat{\imath} + by\,\hat{\jmath} + cz\,\hat{k}$ and σ is the surface of the sphere $x^2 + y^2 + z^2 = 1$ using divergence theorem.
5. Find the general solution of $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$
6. Form an ODE from the basis e^{-2x}, e^{-3x} .
7. Find the Laplace transform of $e^{-2t}\cos^2 t$
8. Find the inverse Laplace transform of $\frac{4s+32}{s^2+16}$
9. Find the Fourier cosine integral of $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$
10. Find the Fourier sine transform of $f(x) = e^{-|x|}$.

PART B
(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Find $div\,\vec{F}$ and $curl\,\vec{F}$ of $\vec{F}(x,y,z) = e^{xy}\hat{\imath} - 2cosy\hat{\jmath} + sin^2 z\hat{k}$ (7)
    b) Show that $\int_{(0,0)}^{(3,2)} 3x^2\,e^y dx + x^3 e^y dy$ is independent of the path and then find its value.   (7)

OR

12. a) Evaluate the line integral $\int_c (xy + z^3)$ ds from (1,0,0) to (-1,0,π) along the helix C that is represented by the parametric equations $x = \cos t$, $y = \sin t$, $z = t$ (7)
    b) Find the work done by the force field $\vec{F} = \frac{1}{x^2+y^2}\hat{\imath} + \frac{4}{x^2+y^2}\hat{\jmath}$ on a particle that   (7)

moves along the curve C , where C is the part of the circle $x^2 + y^2 = 16$ in the first quadrant oriented counter clock wise from (4,0) to (0,4) .

MODULE II

13. a) Evaluate $\oint_C x^2 y\,dx + (y + xy^2)dy$ where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$ using Green's theorem   (7)

    b) Find the mass of the lamina that is the portion of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1 \ and \ z = 3$ if the density function is $\rho(x, y, z) = x^2 z$ (7)

OR

14. a) Use divergence theorem to find the outward flux of the vector field $\bar{F} = (x^2 + y)\hat{\imath} + xy\,\hat{\jmath} - (2xz + y)\hat{k}$ across the surface σ of the tetrahedron bounded by $x + y + z = 2$ and the coordinate planes.   (7)

    b) Use Stoke's theorem to evaluate $\oint_C \bar{F}.d\bar{r}$ where $\bar{F} = 2z\,\hat{\imath} + 3x\,\hat{\jmath} + 5y\,\hat{k}$ and C is the boundary of the paraboloid $z = 4 - x^2 - y^2$ above the XY plane with upward orientation.   (7)

MODULE III

15. a) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-x}cosx,$ using method of undetermined coefficients.   (7)

    b) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 10y = 0$ (7)

OR

16. a) Solve of $\frac{d^2y}{dx^2} + 4y = \cos 2\,x$ using method of variation of parameters.   (7)

    b) Solve the initial value problem
$y'''\text{-}\,y'' + 100y'\text{-}100y = 0, y(0) = 4\,; y'(0) = 11\,, y''(0) = \text{-}299$ (7)

MODULE IV

17. a) Using Laplace transform, solve $y'' + 2y' - 3y = sint, y(0) = y'(0) = 0$ (7)

    b) Using convolution theorem, find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$ (7)

OR

18. a) Find

       i. $L^{-1}\left\{\frac{e^{-2s}}{s+3}\right\}$

       ii. $L^{-1}\left\{\frac{-s+11}{s^2-2s-3}\right\}$ (7)

    b) Using Laplace transform, solve $y'' + 3y' + 2y = \delta(t - 1),$
$y(0) = y'(0) = 0$ (7)

MODULE V

19. a) Find the Fourier integral representation of $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ (7)

b)
Find the Fourier cosine transform of $f(x) = \begin{cases} 1, & 0 < x < 1 \\ -1, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$ (7)

OR

20. a) Find the Fourier transform of $f(x) = \begin{cases} xe^{-x}, & -1 < x < 0 \\ 0, & otherwise \end{cases}$ (7)

b) Find the Fourier sine integral of $f(x) = \begin{cases} x, & 0 < x < a \\ 0, & x > a \end{cases}$ (7)

****