Register No.: ………………………… Name: …………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)
(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)
FOURTH SEMESTER B.TECH DEGREE EXAMINATION (Regular), JULY 2022

(2020 SCHEME)

Course Code : **20CST294**

Course Name: **Computational Fundamentals for Machine Learning**

Max. Marks: **100** **Duration: 3 Hours**

Normal Distribution tables are allowed

PART A

(Answer all questions. Each question carries 3 marks)

1. Is the set W={(1,1,3) , (2,2,6)} linearly dependent? Justify.
2. Define Linear transformation.
3. In a Euclidean inner product, find the Cosine of the angle between the vectors: $\underline{a}$ =(-3,1,0) and $\underline{b}$ =(2,-1,3) that satisfies Cauchy Schwartz inequality.
4. For the function $f(x,y,z) = x^2 + 3y^2 + 2z^2$, find the gradient and its magnitude at (-1,-2,1).
5. Orthogonally project the vector $\binom{2}{3}$ into the line $y = 2x$.
6. Find c if X be a random variable with PDF given by $f_X(x)=\begin{cases} cx^2 & |x|\le 1 \\ 0 & \text{otherwise} \end{cases}$
7. Find all the 1st order partial derivatives of the following function:
   $f(u,v,p,t)= 8u^2t^3p - \sqrt{v}\ p^2 t^{-5}$
8. A fair (unbiased) die is rolled. (i) Find the probability that the number rolled is a five, given that it is odd. (ii) Find the probability that the number rolled is odd, given that it is a five.
9. What is convex optimization? Explain with respect to dual problem.
10. Consider the univariate function $f(x) = x^3 + 6x^2 - 3x - 5$. Find its stationary points and indicate whether they are maximum, minimum, or saddle points.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Solve the following system using Gaussian Elimination: (8)

$$x-2y+z=0$$
$$2x+y-3z=5$$
$$4x-7y+z=-1$$

b) Show that the equations: (6)
$2x + y + z = 5$,
$x + y + z = 4$,
$x - y + 2z = 1$ are consistent and hence solve them.

OR

12. a) Diagonalize the following matrix: (10)

$$\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

b) For what value of c is the set of vectors { (1, c, 1),(0, 1, 1),(1, 0, 2)} linearly dependent? (4)

MODULE II

13. a) If $V=\mathbb{R}^3$ with the Euclidean inner product, apply the Gram-Schmidt algorithm to orthogonalize the basis: (10)
{(1,−1,1),(1,0,1),(1,1,2)}

b) Find the characteristic equation, eigen value of the following matrix: (4)

$$\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

OR

14. a) Find the SVD for the matrix A= (12)

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

b) Find the trace of the matrix (2)

$$\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

MODULE III

15. a) Find the magnitude of the gradient for the function (8)
$(x,y,z)= x^2 +3y^2 +z^3$ at the point (1,1,1).

b) Find all the second partial derivatives of $f(x,y)=x^2y^2e^{2xy}$ (6)

OR

16. a) Compute the gradient of $f(x,y,z)= 1/(x^2+y^2+z^2)$ (8)

b) Find the Taylor series for the function $x^4 - x^2 + 1$ centered at $a= -1$. (6)

MODULE IV

17. a) A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighbourhood have the flu, while the other 10% are sick with measles. Let F stand for an event of a child being sick with flu and M stand for an event of a child being sick with measles. Assume for simplicity that $F \cup M = \Omega$, i.e., that there are no other maladies in that neighbourhood. A well-known symptom of measles is a rash (the event of having which we denote R). Assume that the probability of having a rash if one has measles is $P(R \mid M) = 0.95$. However, occasionally children with flu also develop a rash, and the probability of having a rash if one has flu is $P(R \mid F) = 0.08$. Upon examining the child, the doctor finds a rash. What is the probability that the child has measles? (7)

b) The time, to the nearest whole minute, that a city bus takes to go from one end of its route to the other has the probability distribution shown. As sometimes happens with probabilities computed as empirical relative frequencies, probabilities in the table: (7)

x	42	43	44	45	46	47
P(x)	0.1	0.23	0.34	0.25	0.05	0.02

Find the average time the bus takes to drive the length of its route.

OR

18. a) It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability of a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email? (10)

b) Let X be a normal random variable with mean $\mu = 10$ and standard deviation $\sigma = 2.5$. Compute the following probabilities: (4)

a. $P(X < 14)$.
b. $P(8<X<14)$

MODULE V

19. a) Find the local and global minimizers and maximizers (if they exist) of the functions: $f(x) = x^4 + 4x^3 + 6x^2 + 4x$ (7)

b) A farmer has 500 acres of land to plant corn and soybeans. During the last few years, market prices have been stable and the farmer anticipates a profit of \$900per acre on the corn harvest and \$800 per acre on the soybeans. The farmer must take into account the time it takes to plant and harvest each crop, which is 3 hr/acre for corn and 2 hr/acre for soybeans. If the farmer has at most 1300 hr to plant, care for, and harvest each crop, how many acres of each crop should be planted in order to maximize profits? (7)

OR

20. a) Using Lagrange multipliers, optimize the function $f(x, y) = 81x^2 + y^2$ subject to the constraint $4x^2 + y^2 = 9$. (6)

b) Find the critical points of the function: (8)
$f(x, y) = 2x^3 - 3x^2 y - 12x^2 - 3y^2$

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