Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTERB.TECH DEGREE EXAMINATION (Regular), JULY 2022
COMMON TO EE, EC
(2020 SCHEME)

Course Code: **20MAT204**

Course Name: **Probability, Random Processes and Numerical Methods**

Max. Marks: **100** **Duration: 3 Hours**

Scientific calculators and statistical tables may permitted in the examination hall.

PART A

(Answer all questions. Each question carries 3 marks)

1. The probability mass function of a discrete random variable is given by $f(x) = kx^2$, x=1,2,3,4. Find the value of k.
2. A firm sells four items randomly selected from a large lot that is known to contain 12% defectives. Find the probability that among the four items sold at most one is defective.
3. Find c for which the following is a valid probability density function of a continuous random variable;
   $f(x) = cxe^{-x}, 0 < x < \infty$
4. The time in hours required to repair a machine is exponentially distributed with mean 20. What is the probability that the required time exceeds 30 hr.
5. Define stationary random process. Define two types of stationary random process.
6. Write down the properties of the power spectral density.
7. Solve $x^3 = 25$ by Newton-Raphson method correct to 3 decimal places.
8. Write down the Newton's forward and backward difference interpolation formula.
9. Write the normal equations for fitting a straight line to a given set of pairs of data points.
10. Using Euler's method to solve $\frac{dy}{dx} = x + xy + y\,, y(0) = 1$. Compute y at x = 0.15 by taking h= 0.05.

PART B

(Answer one full question from each module, each question carries 14marks)

MODULE I

11. a) Earthquake occur in a region at an average rate of 5 per year according to a Poisson distribution. What is the probability that; (7)
    (i) No earthquake would occur next year?
    (ii) No earthquake would occur in exactly two of the next five years?

    b) Find the probability that in a family of 4 children there will be; (7)
    (a) At least 1 boy
    (b) 1 or 2 girls
    (c) No girl
    (d) Out of 1000 such families chosen at random how many would you expect to

have at least 1 boy, 1 or 2 girls, No girl.

OR

12. a) The joint distribution (X, Y) is given by p (x, y) = c(x+y), x=0,1,2 ; y=0,1,2,3. (7)
   (i) Find the value of c.
   (ii) Find the marginal distributions of X and Y.
   (iii) Are X & Y independent?

b) The joint probability distribution of two random variables X and Y are given in the following table: (7)

	Y=0	Y=1	Y=2
X=0	0.2	0.1	0.1
X=1	0.3	0.2	0.1

Compute;
(i) E(X)
(ii) E(Y)
(iii) Var(X)
(iv) Var(Y)

MODULE II

13. a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. (7)

b) Derive the mean and variance of uniform distribution. (7)

OR

14. a) If a random variable has probability density

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

Find the probability that it will take on a value; (7)
   (i) between 1 and 3
   (ii) greater than 0.5
   (iii) find the mean and variance of X.

b) In an intelligence test administrated on 1000 children, the average was 60 and standard deviation was 20. Assuming that the marks obtained by the children follow normal distribution, find the number of children who have scored; (7)
   (i) over 90 marks
   (ii) below 40 marks
   (iii) between 50 and 80 marks.

MODULE III

15. a) A random process X(t) is defined by $X(t) = 2\cos(5t + \emptyset)$ where $\emptyset$ is uniformly distributed in $[0, 2\pi]$. Find the mean, autocorrelation and autocovariance. (7)

b) A random process X(t) is defined by $X(t) = a\sin(\omega t + \emptyset)$ where $\emptyset$ is uniformly distributed in $[0, 2\pi]$. Show that X(t) is WSS. (7)

OR

16. a) Let $X(t) = ACos\lambda t + BSin\lambda t$, where A and B are independent random variables with (7)

zero mean and equal variance. Show that X(t) is WSS.

b) If the customer arrive at a counter in accordance with poisson distribution with rate of 2 per minute. Find the probability that the interval between two consecutive arrivals; (7)

(i) more than one minute
(ii) between 1 minute and 2 minutes.
(iii) Less than or equal to 4 minutes

MODULE IV

17. a) Use Newton-Raphson method to find a root of the equation (7)

$$x^3 - 2x - 5 = 0$$

b) Using Newton's forward difference formula find the interpolating polynomial for the following data; (7)

x	0	1	2	3
y	0	2	6	18

OR

18. a) The population of a city for various years as per census records is as follows; (7)

Year	1971	1981	1991	2001	2011
Population(in lakhs)	20	24	28	32	45

Apply Lagrange's interpolation method to estimate the population of the city during the year 2005.

b) Evaluate $\int_0^2 xe^x dx$ using Simpson's rule with 8 subintervals. Compare the result with actual value. (7)

MODULE V

19. a) Using Gauss-Seidel method, solve the following system of equations; (7)

$$8x_1 + x_2 + x_3 = 8$$
$$2x_1 + 4x_2 + x_3 = 4$$
$$x_1 + 3x_2 + 5x_3 = 5$$

b) Fit a curve of the form $y = a + bx^2$ to the following data: (7)

x	1	2	3	4	5	6
y	0.56	0.89	1.04	1.63	2.95	4.5

OR

20. a) Solve the system of equations using Jacobi's method: (7)

$$27x_1 + 6x_2 - x_3 = 85$$
$$x_1 + x_2 + 54x_3 = 110$$
$$6x_1 + 15x_2 + 2x_3 = 72$$

b) Given the initial value problem $\frac{dy}{dx} = \sqrt{x+y}$, $y(0) = 1$, use Runge-Kutta method of fourth order to find $y(0.2)$ with h=0.1 (7)

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