Register No.: ……………………………… Name: ……………………………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

SECOND SEMESTERB.TECH DEGREE EXAMINATION (Regular), JULY 2022

(2020 SCHEME)

Course Code: **20MAT102**

Course Name: **Vector Calculus, Differential Equations and Transforms**

Max. Marks: **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the velocity, speed and acceleration at the given time t of a particle moving along the curve $\vec{r}(t) = t\hat{\imath} + \frac{1}{2}t^2\hat{\jmath} + \frac{1}{3}t^3\hat{k};\ t = 2$.
2. If $\phi(x, y, z) = xsinz + ysinx + zsiny$ is a potential function of $\overrightarrow{F}$, find $\overrightarrow{F}$
3. Using Green's theorem evaluate $\int_C\ tan^{-1}ydx - \frac{y^2x}{1+y^2}dy,$ where C is the square with vertices $(0,0), (1,0), (1,1)\ and\ (0,1)$
4. Use Divergence theorem to evaluate $\iint_\sigma\ F.ndS$ where $F = (x - z)i + (y - x)j + (2z - y)k$ where σ is the surface of the cylindrical solid bounded by $x^2 + y^2 = a^2, z = 0$ and $z = 1$
5. Find a general solution of $x^2y'' - xy' - 3y = 0$
6. Solve $y^{iv} - 81y = 0$
7. Find $\mathcal{L}[t^2\ u(t - 1)]$
8. Find $\mathcal{L}^{-1}\left[\frac{1}{s(s^2+a^2)}\right]$
9. Find the Fourier sine transform of $e^{-|x|}$.
10. Does Fourier sine transform of $e^x, 0 < x < \infty$ exist? Justify your answer

PART B

(Answer one full question from each module, each question carries 14marks)

MODULE I

11. a) Calculate the directional derivative of $F(x,y,z)=x^2y-yz^3+z$ at the point P(-1,2,0) in the direction of $\vec{u} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ (7)

    b) Show that the integral $\int_c\ 2xe^{-y}dx + (2y - x^2e^{-y})dy$ is independent of path. Hence evaluate the integral from (1,0) to (4,1). (7)

OR

12. a) Determine the work done by the force field $F = (x + y)i + xyj - z^2k$ on a particle that moves along the line segments from(0,0,0)to (1,3,1) to $(2, -1,5)$ (7)

b) Find $div\,\vec{F}$ and $curl\,\vec{F}$ at the point $(1,-1,1)$ where $\vec{F} = xz^3\hat{\imath} - 2x^2yz\hat{\jmath} + 2yz^4\hat{k}$ (7)

MODULE II

13. a) Using Green's theorem to evaluate the line integral $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $y = \sqrt{x}, y = x^2$ and oriented counter clockwise (7)

b) Use Stoke's theorem to evaluate $\int_C \vec{F}.dV$ where $\vec{F} = xy\hat{\imath} + yz\hat{\jmath} + xz\hat{k}$; C is the triangle in the plane $x + y + z = 1$ with vertices $(1,0,0), (0,1,0)$ and $(0,0,1)$ with a counter clockwise orientation looking from the first octant towards origin. (7)

OR

14. a) Evaluate the surface integral $\iint_\sigma f(x,y,z)dS$ where $f(x,y,z) = z^2$ and σ is the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$and $z = 3$ (7)

b) Determine whether the following vector field F is free of sources and sinks. If not locate them (7)

i. $F = x^3i + y^3j + 4z^3k$

ii. $F = 2(x^3 - 2x)i + 2(y^3 - 2y)j + 2(z^3 - 2z)k$

MODULE III

15. a) Solve using the method of undetermined coefficients $(D^2 - 16I)y = 9.6e^{4x} + 30e^x$ (7)

b) Solve using the Method of variation of parameters $y'' - 4y' + 5y = e^{2x}cosecx$ (7)

OR

16. a) Solve $y'' + 4y' + (\pi^2 + 4)y = 0, y\left(\frac{1}{2}\right) = 1,\ y'\left(\frac{1}{2}\right) = -2$ (7)

b) Solve $y'' + 2y' + 0.75y = 2cosx - 0.25sinx$ (7)

MODULE IV

17. a) Evaluate (i) $\mathcal{L}\left[\frac{cos2t - cos3t}{t}\right]$ (7)

(ii) $\mathcal{L}\left[e^{-4t}\int_0^t t\sin(3t)\,dt\right]$

b) Find (i) $\mathcal{L}^{-1}[cot^{-1}(1+s)]$ (7)

(ii) $\mathcal{L}^{-1}\left[\frac{3s-2}{s^2+2s+6}\right]$

OR

18. a) Use Convolution theorem to find the inverse Laplace transform of $\frac{s}{(s-1)(s^2+4)}$ (7)

b) Using Laplace transform solve $y'' + 5y' + 6y = e^{-2t}$ given $y(0) = y'(0) = 1$ (7)

MODULE V

19. a) Find the Fourier sine transform of

$$f(x) = \begin{cases} x, 0 < x < 1 \\ 2 - x, 1 < x < 2 \\ 0, x > 1 \end{cases}$$ (7)

b) Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 2, & if\ |x| < 1 \\ 0, & if\ |x| > 1 \end{cases}$$ (7)

OR

20. a) Find the Fourier cosine transform of $f(x) = \begin{cases} 1 - x^2, & if\ 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$. (7)

Hence show that $\int_0^\infty \left(\frac{sinw - wcosw}{w^3}\right) \cos({}^W\!/_2) dw = \frac{3\pi}{16}$

b) Find the Fourier sine integral representation of $f(x) = \begin{cases} sinx, & if\ 0 \le x \le \pi \\ 0, & if\ x > \pi \end{cases}$ (7)

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