G 592

(Pages : 4)

Reg. No...............................

Name....................................

B.TECH. DEGREE EXAMINATION, MAY 2014

First and Second Semesters

EN 010 101—ENGINEERING MATHEMATICS—I

(New Scheme—2010 admission onwards—Regular/Improvement/Supplementary)

[Common for all Branches]

Time : Three Hours

Maximum : 100 Marks

Part A

Answer **all** *questions.*
Each question carries 3 *marks.*

1. Find the characteristic roots of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

2. If $x = r\cos\theta, y = r\sin\theta$, show that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$.

3. Evaluate $\int_0^1 \int_x^{\sqrt{x}} \left(x^2 + y^2\right) dy\, dx$.

4. Solve $\frac{dy}{dx} + \frac{x - 2y}{2x - y} = 0$.

5. Find the Laplace Transform of $e^{-3t}\left(\cos 4t + 3\sin 4t\right)$.

(5 × 3 = 15 marks)

Part B

Answer **all** *questions.*
Each question carries 5 *marks.*

6. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 4 & 8 & 0 \end{bmatrix}$.

7. If $u = 2xy,\ v = x^2 - y^2, y = r\sin\theta, x = r\cos\theta$, evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$.

Turn over

8. Change the order of integration in $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dy\,dx$.

9. Solve $\left(x^2+y^2\right)dx - 2xy\,dy = 0$.

10. Find $L\left\{\int_0^t \frac{\sin x}{x}\,dx\right\}$.

(5 × 5 = 25 marks)

Part C

Answer **all** *questions.*
Each full question carries 12 *marks.*

11. (a) Using matrices, solve the system of the equations :

$$x+y+z=9$$
$$2x+5y+7z=52$$
$$2x+y-z=0.$$

(7 marks)

(b) Verify that the matrix $\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ is orthogonal. (5 marks)

Or

12. Reduce the sum of squares to the quadratic forms :

$$9x^2 - 6y^2 - 8z^2 + 6xy - 14xz + 18yz + 12yw - 4zw.$$

(12 marks)

13. (a) If $u = \log_e\left(x^3+y^3+z^3-3xyz\right)$, prove that :

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}.$$

(5 marks)

(b) If $v = \log_e \sin\left\{\frac{\pi\left(2x^2+y^2+xz\right)^{1/2}}{2\left(x^2+xy+2yz+z^2\right)^{1/3}}\right\}$, find the value of $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}+z\frac{\partial v}{\partial z}$ when $x=0$, $y=1$, $z=2$.

Or

(7 marks)

14. (a) If $r=\sqrt{x^2+y^2}, \theta=\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{\partial(r,\theta)}{\partial(x,y)}=\frac{1}{\sqrt{x^2+y^2}}$. (6 marks)

(b) If $xyz = 8$, find the values of x, y for which $u=\frac{5xyz}{(x+2y+4z)}$ is a maximum. (6 marks)

15. (a) Evaluate $\iint r\sin\theta\, dr\, d\theta$ over the Cardioid $r=a(1-\cos\theta)$ above the initial line. (6 marks)

(b) Transform the following to Cartesian form and hence evaluate $\int_0^{\pi}\int_0^{a} r^3\sin\theta\cos\theta\, dr\, d\theta$.

(6 marks)

Or

16. (a) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1}\frac{dz\, dy\, dx}{\sqrt{x^2+y^2+z^2}}$. (6 marks)

(b) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$. (6 marks)

17. (a) Find the complete solution of :

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=\sin 3x\cos 2x.$$ (7 marks)

(b) Solve $\frac{dy}{dx}=(4x+y+1)^2$ if $y(0)=1$. (5 marks)

Or

18. (a) Solve $3e^x\tan y\, dx+(1-e^x)\sec^2 y\, dy=0$. (5 marks)

(b) Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-3y=x^3$. (7 marks)

19. (a) Find the inverse Laplace transform of $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$. (5 marks)

(b) Solve by the Laplace transform method, $y'''+2y''-y'-2y=0$ given $y(0)=y'(0)=0$ and $y''(0)=6$.

(7 marks)

Or

20. (a) Find the Laplace transform of the square wave function of period defined as :

$$f(t) = 1 \text{ when } 0 < t < k/2$$
$$= -1 \text{ when } k/2 < t < k.$$

(6 marks)

(b) Using second shifting property, find the Laplace transform of :

$$f(t) = \begin{cases} t - 1 (1 < t < 2) \\ 3 - t (2 < t < 3) \end{cases}.$$

(6 marks)

[5 × 12 = 60 marks]