Register No.: ……………………………… Name: ………………………………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

THIRD SEMESTER B.TECH DEGREE EXAMINATION (S), MAY 2022

COMMON TO CE,CH,EC,EE,FT,ME,RA

(2020 SCHEME)

Course Code : **20MAT201**

Course Name: **Partial Differential Equations and Complex Analysis**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Form a Partial differential equation from the relation $z = f(x^2 + y^2) + x + y$
2. Solve $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y} + a$.
3. What are the three possible solutions of one dimensional wave equation?
4. State any three assumptions in deriving the one dimensional heat equation .
5. Show that $f(z) = \begin{cases} \frac{Re(z)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is discontinuous at z=0
6. Define critical point and fixed point of a complex function $f(z)$.
7. Evaluate $\oint \ln z \, dz$ where C is the unit circle $|z| = 1$.
8. Expand $f(z) = \frac{\sin z}{z-\pi}$ as Laurent's series about $z = \pi$.
9. Determine the nature and type of singularity of $z \sin(\frac{1}{z})$.
10. Find the residue of the function $\frac{1-e^{2z}}{z^4}$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Solve $p - 2q = 3x^2 \sin(y + 2x)$. (7)

    b) Solve $p = (qy + z)^2$. (7)

OR

12. a) Find the differential equation of all spheres whose centers lie on the z-axis. (7)

    b) Using the method of separation of variables solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, given $u(x, 0) = 4\, e^{-x}$. (7)

MODULE II

13. a) Find the D' Alembert's solution of the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x) = k(\sin x - \sin 2x)$. (6)

b) An insulated rod of length l has its ends A and B are maintained at $0^0\ C$ and $200^0 C$ C respectively under steady state condition prevails. If the temperature at B is suddenly reduced to $0^0\ C$ and maintained at $0^0\ C$. Find the temperature at a distance x from A at time t. (8)

OR

14. a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving to each of its points a velocity $\lambda x(l - x)$, find the displacement of the string at any distance x from one end at any time t. (7)

b) Derive the solution of one-dimensional heat equation (7)

MODULE III

15. a) Prove that $w = \cos hz$ is an entire function . Also find its derivative. (6)

b) Discuss the mapping $w = z^2$. (8)

OR

16. a) Find the analytic function whose real part is $cosx \cos hy$. (7)

b) Under the transformation $w = \frac{1}{z}$ find the image of $|z - 2i| = 2$. (7)

MODULE IV

17. a) Evaluate $\int_0^{1+i}(x^2 - ixy)\, dz$ along the path $y = x^2$. (7)

b) Evaluate $\int_C \frac{z+4}{z^2+2z+5}\, dz$ where C is $|z + 1 - i| = 2$. (7)

OR

18. a) Using Cauchy's integral formula evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)}\, dz$ where C is $|z - 2| = 2$. (7)

b) Find the Maclaurin series expansion of $\frac{z+2}{1-z^2}$. (7)

MODULE V

19. a) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ in Laurent's series about $z = -2$ in $0 < |z + 2| < 1$. (7)

b) Evaluate $\int_C \frac{\cos \pi z}{z^2-1}\, dz$ using Cauchy's residue theorem where C is the rectangle with vertices $2 \pm i, -2 \pm i$. (7)

OR

20. a) Evaluate $\int_C \frac{e^z}{(z+1)^3}\, dz$ using Cauchy's residue theorem where C is $|z + 1| = 2$ (7)

b) Using contour integration evaluate $\int_0^{\pi} \frac{1}{5-3\sin\theta}\, d\theta$. (7)

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