Register No.: ……………………………… Name: ……………………………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER B.TECH DEGREE EXAMINATION (S), MAY 2022

COMMON TO ALL BRANCHES

(2020 SCHEME)

Course Code : **20MAT101**

Course Name: **Linear Algebra and Calculus**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ by reducing it to the row echelon form.
2. Find the eigen values of the matrix $A = \begin{bmatrix} 7 & -1 \\ 4 & 3 \end{bmatrix}$.
3. Let $f(x,y) = \sqrt{3x+2y}$ then find the slope of the surface $z = f(x,y)$ in the x-direction and y-direction at the point (2,5).
4. Compute the total differential dw of the function $w = \tan^{-1}(xyz)$.
5. Evaluate $\int_{\frac{\pi}{2}}^{\pi} \int_{1}^{2} x \sin(xy)\, dy\, dx$.
6. Find the area of the plane region enclosed by the parabola $y = x^2$ and the line $y = 2x + 3$.
7. Find the rational number represented by 0.784784784 … .
8. Test the convergence of the series $\sum_{k=1}^{\infty} \left(1 + \frac{3}{k}\right)^k$.
9. Find the Taylor series expansion of $\frac{1}{x+2}$ about $x = 1$.
10. Find the Euler coefficients of $f(x) = x^2; -\pi < x < \pi$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Solve the following system of equations by Gauss elimination and back substitution method: $3x_1 + 3x_2 + 2x_3 = 1,\ x_1 + 2x_2 = 4,\ 10x_2 + 3x_3 = -2,\ 2x_1 - 3x_2 - x_3 = 5$. (7)

    b) Find an orthogonal matrix which diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. (7)

OR

12. a) What kind of conic section is given by the quadratic form $Q = 4{x_1}^2 + 6x_1x_2 - 4{x_2}^2 = 10$? Transform it to principal axes. Express $x^T = [x_1, x_2]$ in terms of the new coordinate vector $y^T = [y_1, y_2]$. (7)

    b) For what values of a and b do the equations (7)

$x + 2y + 3z = 6, x + 3y + 5z = 9, 2x + 5y + az = b$ have
(i) no solution (ii) a unique solution (iii) more than one solution.

MODULE II

13. a) Let $w = \ln(3x^2 - 2y + 4z^3)$; $x = t^{1/2}$, $y = t^{2/3}, z = t^{-1}$. Use chain rule to find $\frac{dw}{dt}$ at $t = 1$. (7)

b) Locate all relative maxima, relative minima and saddle points of $f(x, y) = xy + \frac{2}{x} + \frac{4}{y}$. (7)

OR

14. a) Find the local linear approximation $L(x, y, z)$ to $f(x, y, z) = \frac{x+y}{y+z}$ at the point $P(-1, 1, 1)$. Compare the error in approximating f by L at the specified point $Q(-0.99, 0.99, 1.01)$. (7)

b) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$. (7)

MODULE III

15. a) Evaluate $\iint_R (x^2 + y^2)dydx$ over the region in the positive quadrant for which $x + y \leq 1$. (7)

b) Evaluate $\int_0^\infty \int_0^\infty \frac{e^{-y}}{y} dydx$ by changing the order of integration. (7)

OR

16. a) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (7)

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical polar co-ordinates. (7)

MODULE IV

17. a) Determine whether the series $2 + \frac{2}{5} + \frac{2}{5^2} + \frac{2}{5^3} + \ldots\ldots$ converges and if so find the sum. (7)

b) Test the convergence of the series (7)

(i) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{5k}{3^k}$ (ii) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{8k^2 - 3k}}$

OR

18. a) Test the convergence of the series (7)

(i) $\sum_{k=1}^{\infty} \frac{x^k}{2^k k^2}$; $x > 0$ (ii) $\sum_{k=1}^{\infty} \frac{\ln k}{k}$

b) Check whether the series absolutely convergent, conditionally convergent or divergent. (7)

(i) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3^{2k-1}}{k^2+1}$ (ii) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k(k+1)}}$

MODULE V

19. a) Obtain the Fourier series to represent $f(x) = \frac{1}{4}(\pi - x)^2$ in the interval $0 < x < 2\pi$ with $f(x + 2\pi) = f(x)$. Hence deduce that $\sum \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$. (7)

b) Find the Binomial series of $\frac{1}{(1+x)^2}$. (7)

OR

20. a) Find the Fourier sine series for unity in the interval $(0, \pi)$ and hence show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots = \frac{\pi^2}{8}$. (7)

b) Obtain the Fourier series expansion of the function $f(x) = \begin{cases} -\pi \, ; when - \pi < x < 0 \\ x \, ; when \;\; 0 < x < \pi \end{cases}$. Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots = \frac{\pi^2}{8}$. (7)

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