Register No.: ……………………………… Name: …………………………………………………………….

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

THIRD SEMESTER B. TECH DEGREE EXAMINATION (Regular), FEBRUARY 2022
COMPUTER SCIENCE AND ENGINEERING
(2020 SCHEME)

Course Code: **20MAT203**

Course Name: **Discrete Mathematical Structures**

Max. Marks: **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Define tautology and contradiction.
2. Use Truth table to verify Law of syllogism.
3. In how many ways can the letters of the word 'ARRANGE' be arranged such that the two R's do not occur together.
4. Show that if any five numbers are selected from S={1,2,…,8} at least two of them will add up to 9.
5. Define a Poset. Explain with example.
6. Determine whether the function f: $R \to R$ defined by f(x)= e^{x^2} is one to one. Determine its range.
7. What is meant by exponential generating function? Find the exponential generating function of the sequence 1,-1,1,-1 …
8. Solve the recurrence relation $a_n=7a_{n-1}$ where $n\geq 1$ *and* $a_2 = 98$
9. Give an example of a Semi-group which is not a monoid.
10. Prove that every cyclic group is abelian.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Check the validity of the following argument (7)

$$\begin{array}{c} p \to r \\ r \to s \\ t \vee \neg s \\ \neg t \vee u \\ \hline \therefore \neg u \to \neg p \end{array}$$

b) Let the universe be the set of all real numbers, consider the open statements (7)
$p(x): 3x-7=20$, $q(x): 3x=27$, $r(x): x=9$
Check the validity of the following
If $3x-7=20$ then $3x = 27$.
If $3x=27$, then $x=9$.
Therefore, if $3x-7=20$ then $x=9$.

OR

12. a) Check whether the propositions $(q \wedge \neg r)$ and $p \vee (q \wedge \neg r)$ are logically equivalent or not. (7)

b) Let the universe set be the entire student body at a particular college. One specific student, Mary will be designated by m. Let j(x): x is a junior. S(x): x is senior. p(x): x is enrolled in physical education class. Check the validity of the following argument (7)
No Junior or senior is enrolled in physical education class.
Mary is enrolled in physical education class.
Mary is not a senior.

MODULE II

13. a) A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of (7)
(i) exactly 3 girls.
(ii) at least 3 girls.
(iii) at most 3 girls.

b) Determine the number of positive integers $1 \leq n \leq 10000$ where n is not divisible by 5,6,8 (7)

OR

14. a) How many solutions are there to $x_1 + x_2 + x_3 = 17$ where $x_i \leq 7$ for $1 \leq i \leq 3$ (7)

b) Determine the coefficient of (7)
(i) xyz^2 in $(x+ y+ z)^4$
(ii) xyz^2 in $(w-x+ y+ z)^4$
(iii) xyz^2 in $(2x- y- z)^4$

MODULE III

15. a) If A= {1,2,3} and P(A) be its power set of A. The relation $\subseteq$ be the subset relation defined on the power set. Show that (P(A), $\subseteq$) is a lattice. Draw the Hasse diagram of (P(A),$\subseteq$). (7)

b) Define a complemented Lattice. Check whether (D_{42}, /) is a complemented lattice or not. (7)

OR

16. a) If f, g and h are functions of integers such that $f(n)=n^2$, $g(n)=(n+1)$ and $h(n)= n-1$ find (i) f o g o h (ii) g o f o h (iii) h o f o g (6)

b) If R is a relation in the set of all integers such that R= {(x, y) /x, y$\in Z$, x-y is divisible by 3}. Prove that R is an equivalence relation. Describe the distinct equivalence classes of Z induced by R. (8)

MODULE IV

17. a) Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ where $n \geq 2$, $a_0 = -1$, $a_1 = 8$ (7)

b) A bank pays 6% annual interest on savings compounding the interest monthly. If Roy deposits Rs 2000/- on the first day of May, how much will this deposit worth a year later. (7)

OR

18. a) Solve the recurrence relation $a_{n+2} = 4a_{n+1} - 4a_n, n \geq 0$, $a_0 = 1$ $a_1 = 3$ (6)

b) Solve the recurrence relation $a_{n+2} - 8a_{n+1} + 16a_n = 8(5)^n; n \geq 0,$ $a_0 = 12, a_1 = 5$ (8)

MODULE V

19. a) Show that any group G is abelian if and only if $(ab)^2 = a^2 b^2$ for all $a, b \in G$ (6)

b) If X = {1,2,3}. List all permutations S_3 of X and prove that (S_3, o) is a group where the operation o is the composition of permutations. Is it abelian? (8)

OR

20. a) State and prove Lagrange's Theorem? (6)

b) Show that Q^+ of all positive rational numbers from an abelian group under the operation * defined by $a*b = \frac{1}{2}ab$; $a, b \in Q^+$ (8)

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