

Register No.: Name:

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH DEGREE EXAMINATION (Regular), FEBRUARY 2022*(Telecommunication Engineering)***(2021 Scheme)****Course Code: 21TE101****Course Name: Applied Linear Algebra****Max. Marks: 60****Duration: 3 Hours****PART A***(Answer all questions. Each question carries 3 marks)*

1. Explain the algebraic structures ring and field.
2. Solve the linear system

$$\begin{aligned} x + y &= 4 \\ 3x + 3y &= 6 \end{aligned}$$
3. Find the matrix representation of the linear map $T: \mathbb{R}^2$ to \mathbb{R}^3 defined by $T(x,y) = (2x-5y, 3x+y)$ relative to the basis $\{u_1=(2,1), u_2=(3,2)\}$ of \mathbb{R}^2 .
4. State Pythagoras theorem and verify the theorem for the following orthogonal set $u=(1,2,-3,4)$, $v=(3,4,1,-2)$ and $w=(3,-2,1,1)$.
5. Explain positive and negative definite matrices.
6. Check whether the given matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ is diagonalizable or not.
7. Explain about singular values of a matrix.
8. Determine the null space of the given matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$

PART B*(Answer one full question from each module, each question carries 6 marks)***MODULE I**

9. a) Show that the three vectors $v_1=(1,2,2,-1)$, $v_2=(4,9,9,-4)$, $v_3=(5,8,9,-5)$ are linearly independent. (3)
- b) Express the vector $v = (9,2,7)$ as a linear combination of vectors $u=(1,2,-1)$ and $v=(6,4,2)$. (3)

OR

10. a) Determine whether the vector $v = (3,-2,-4,1)$ is in the span of $S = \{v_1=(1,2,3,4), v_2=(1,-1,1,-1), v_3=(2,0,3,1)\}$ (3)
- b) Show that the vectors $(1,-1,1)$, $(0,1,2)$, $(3,0,-1)$ form a basis for \mathbb{R}^3 . (3)

MODULE II

11. Given a system of equation

$$-mx + y = A$$

$$-nx + y = B$$

(6)

where m, n, A, B are constants.

i) Show that the system will have a unique solution if $m \neq n$.

ii) Show that if $m = n$, then the system will be consistent only if $A = B$.

OR

12. Solve the following system using Gauss Elimination method.

$$x - 2y + z = 0$$

$$2x + y - 3z = 5$$

$$4x - 7y + z = -1$$

(6)

MODULE III

13. Check whether the
- $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$
- is linear transformation or not.

$$T \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \right\} \rightarrow \begin{bmatrix} x - y \\ x + y \\ 2x \end{bmatrix}$$

(6)

OR

14. Find the fundamental subspaces of the matrix
- $A = \begin{bmatrix} 1 & 5 & 3 & 7 \\ 2 & 0 & -4 & -6 \\ 4 & 7 & -1 & -2 \end{bmatrix}$

(6)

MODULE IV

15. Apply the Gram-Schmidt process to transform the basis vectors
- $u_1 = (1, 1, 1)$
- ,
- $u_2 = (0, 1, 1)$
- ,
- $u_3 = (0, 1, 1)$
- into an orthogonal basis
- (v_1, v_2, v_3)
- and then normalize the orthogonal basis vectors to obtain an orthonormal basis
- (q_1, q_2, q_3)
- .

OR

16. a) Show that vectors
- $\{(v_1, v_2, v_3)\}$
- are orthonormal basis of
- \mathbb{R}^3
- where
- $v_1 = \left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right)$
- ,
- $v_2 = \left(\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$
- ,
- $v_3 = \left(\frac{-1}{\sqrt{66}}, \frac{-4}{\sqrt{66}}, \frac{7}{\sqrt{66}} \right)$
- .
-
- b) Determine the inner product of the vectors
- $u_1 = (1, 2, 3)$
- ,
- $u_2 = (2, -3, 4)$
- .

(4)

(2)

MODULE V

17. Diagonalize the given matrix
- $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(6)

OR

18. Given that 2 is an eigen value of $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 1 & 0 \end{bmatrix}$. Determine its geometric multiplicity and give a basis for the associated eigen space. (6)

MODULE VI

19. Determine the SVD of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ (6)

OR

20. Determine the Pseudo inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$ using singular value decomposition method. (6)
