Register No.: ………………………………. Name: ……………………………………………………………..

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH DEGREE EXAMINATION (Regular), FEBRUARY 2022

(Power Systems)

(2021 Scheme)

Course Code : **21PS101**

Course Name: **Applied Mathematics**

Max. Marks : **60** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the Z-transform of a^n.
2. Find the extremals of the functional $\int_a^b (y^2 - y'^2)dx$.
3. Show that $y(x) = e^x(2x - \frac{2}{3})$ is a solution of the Fredholm integral equation $y(x) + 2\int_0^1 e^{x-t}y(t)dt = 2xe^x$.
4. Define point estimator. What are the desirable properties of a good estimator?
5. Give the normal equations for a straight line $y = a + bx$ using method of least squares.
6. Derive Crank – Nicolson formula for the one-dimensional heat equation.
7. Show that the vectors $(1,1,2,4)$, $(2,-1,-5,2),(1,-1,-4,0),(2,1,1,6)$ are linearly dependent.
8. Let T and U be the linear operators on R^2 defined by $T(x_1, x_2) = (-x_2, x_1)$ and $U(x_1, x_2) = (0, x_2)$. Find the transformations $U + T, UT$ and T^2.

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. Find the inverse Z-transform of $\frac{4z^2-2z}{z^3-5z^2+8z-4}$ by method of partial fractions. (6)

OR

10. Solve using Z-transform , $y_{n+2} - 4y_n = 0$, given that $y_0 = 0, y_1 = 2$. (6)

MODULE II

11. Show that the geodesics on a plane are *straight lines*. (6)

OR

12. Find the extremal of the functional $\int_0^\pi (y'^2 - y^2)dx$ under the conditions $y(0) = 0$ and $y(\pi) = 1$ and subject to the constraint $\int_0^\pi y dx = 1$. (6)

MODULE III

13. Solve the Volterra integral equation $y(x) = \frac{1}{6}\int_0^x (x-t)^3 y(t)dt$ by the Transform method. (6)

OR

14. Solve the Volterra integral equation $y(x) = 1 + \int_0^x y(t)dt$ by the successive approximation method. (6)

MODULE IV

15. Find the Maximum Likelihood Estimator of μ and σ^2, using the random sample $x_1, x_2, \dots, x_n$ taken from the normal population $N(\mu, \sigma)$. (6)

OR

16. If x_1, x_2, x_3, x_4 are independent observations from a population with mean μ and variance σ^2. Compare the efficiencies of $t_1 = \frac{2x_1+x_2}{3}, t_2 = \frac{2x_1+3x_2}{5}, t_3 = \frac{x_1+x_2+x_3+x_4}{4}$. Identify the more efficient estimator. (6)

MODULE V

17. Use the method of least squares to fit an equation of the form $y = ax + b$ to the following data (6)

$$y(1) = 6, y(2) = 7, y(3) = 9, y(4) = 10, y(5) = 12$$

OR

18. Fit a parabola of the form $y = ax^2 + bx + c$ to the following data. (6)

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

MODULE VI

19. Show that the vectors $\alpha_1 = (1,0,1), \alpha_2 = (1,2,1), \alpha_3 = (0,1,1)$ form a basis for R^3. Express each of the standard basis vectors as linear combinations of α_1, α_2 and α_3. (6)

OR

20. Define an inner product space. Show that $< x, y > = x_1y_1 - x_2y_1 - x_1y_2 + 4x_2y_2$ defines an inner product in R^2. (6)

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