Register No.: .................................. Name: ..............................................................

# SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

**FIRST SEMESTER M.TECH DEGREE EXAMINATION (Regular), FEBRUARY 2022**

***(TELECOMMUNICATION)***

**(2021 Scheme)**

**Course Code:** **21TE102**

**Course Name:** **Random Processes and Applications**

**Max. Marks:** **60** **Duration: 3 Hours**

***Can use statistical tables if necessary.***

## PART A

***(Answer all questions. Each question carries 3 marks)***

1. In a certain group of computer personnel, 65% have insufficient knowledge of hardware, 45% have inadequate idea of software and 70% are in either one or both of the two categories. What is the percentage of people who know software among those who have a sufficient knowledge of hardware?
2. Show that the random variables X and Y with joint probability density function $f(x,y) = \begin{cases} xye^{-\frac{1}{2}(x^2+y^2)} & x>0, y>0 \\ 0 & otherwise \end{cases}$ are independent.
3. Let $X$ be a continuous random variable with the probability density function $f(x) = \begin{cases} K & if -1 < x < 1 \\ 0 & otherwise \end{cases}$
   Find the value of $K$ and find the mean of $X$.
4. Define Markov-Process.
5. Define a WSS process.
6. For the process $\{X(t): t \geq 0\}, X(t)$ is given by $X(t) = a\cos\theta t + b\sin\theta t$ where $a$ and $b$ are two independent normal variables with $E(a) = E(b) = 0$ and $Var(a) = Var(b) = \sigma^2$ and $\theta$ is a constant. Obtain the mean and variance of the process.
7. Consider a constant random process $X(t) = C$, where $C$ is a random variable with mean $\mu$ and variance $\sigma^2$. Examine whether $X(t)$is mean ergodic.
8. Define Correlation Ergodic process.

## PART B

***(Answer one full question from each module, each question carries 6 marks)***

### MODULE I

9. Let $X$ be a random variable such that
   $P(X=-2) = P(X=-1) = P(X=1) = P(X=2)$ and
   $P(X<0) = P(X=0) = P(X>0)$ (6)
   Determine the probability mass function and the distribution of $X$.

**OR**

10. A random variable $X$ has the density function
$f(x) = K\dfrac{1}{1+x^2}$ where $-\infty < x < \infty$ (6)
Determine $K$ and the distribution function. Evaluate the probability $P(X \geq 0)$.

**MODULE II**

11. Let $X$ and $Y$ be two independent uniform variables over (0,1). Show that the random variables $U = \cos(2\pi x)\sqrt{-2\ln Y}$ and $V = \sin(2\pi x)\sqrt{-2\ln Y}$ are independent standard normal random variables. (6)

**OR**

12. Let $(X, Y)$ be a two-dimensional continuous random variable with joint probability density function $f_{X,Y}(x, y)$. Let $Z=X+Y$. Find the probability density and the distribution functions of $Z$. What do you get when $X$ and $Y$ are independent? (6)

**MODULE III**

13. Find the moment generating function of the Poisson distribution. Also find the first and second moments. (6)

**OR**

14. If $X$, $Y$ and $Z$ are uncorrelated random variables with zero mean and standard deviation 5, 12 and 9 respectively, and $U=X+Y$ and $V=Y+Z$, find the correlation coefficient between $U$ and $V$. (6)

**MODULE IV**

15. Let $X(t)$ be a Poisson process with rate $\lambda$. Then show that $X(t)$ is a Markov process. (6)

**OR**

16. State and prove Chapman-Kolmogorov Theorem. (6)

**MODULE V**

17. A random variable X has the density function $e^{-x}, x \geq 0$. Show that Chebychev's inequality gives $P[|X-1| > 2] < \frac{1}{4}$ and show that actual probability is $e^{-3}$. (6)

**OR**

18. State and prove Chebychev's Inequality. (6)

**MODULE VI**

19. A random process is defined as $X(t) = A\cos\omega t + B\sin\omega t$, where $A$ and $B$ are random variables with $E(A) = E(B) = 0$, $E(A^2) = E(B^2)$ and $E(AB) = 0$. Show that the process $X(t)$ is mean ergodic. (6)

**OR**

20. State and prove Mean-Ergodic Theorem for a random process $X(t)$ to be mean ergodic. (6)

****************************************************