Register No.: ……………………………… Name: ……………………………………………………………

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTERB.TECH DEGREE EXAMINATION (Regular), FEBRUARY 2022
COMMON TO ALL BRANCHES
(2020 SCHEME)

Course Code: **20MAT101**

Course Name: **Linear Algebra and Calculus**

Max. Marks: **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & -1 \\ 2 & -1 & 0 \end{bmatrix}$
2. Reduce the quadratic form $-3{x_1}^2 - 3{x_2}^2 - 3{x_3}^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$ into the principal axes form.
3. Find $\frac{dz}{dx}$ at (1,0) if $z = x^2 + 2xy + 4y^2$ and $y = e^{ax}$.
4. Calculate $f_x(1,2)$ and $f_y(1,2)$ where $f(x,y) = \sqrt{x^2 + 4y^2}$.
5. Evaluate $\iint_R e^{-(x^2+y^2)}\, dA$, where R is the region enclosed by the circle $x^2 + y^2 = 1$.
6. Evaluate $\int_0^1 \int_0^2 \int_0^3 xyz\ dx\, dy\, dz$.
7. Determine the rational number representing the decimal number 6.242424 ...
8. Test the convergence of $\sum_{k=1}^{\infty} \frac{1}{2^k+1}$.
9. Find the Taylor series for $f(x) = \cos x$ about $x = \frac{\pi}{2}$.
10. Obtain the half range Fourier sine series expansion of $f(x) = c$ in $(0, \pi)$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Consider the system of equations

$$\begin{aligned} x + y + z &= l \\ 3x + 4y + 5z &= m \\ 2x + 3y + 4z &= k \end{aligned}$$

Show that the given system of equations has no solution if $l = m = k = 1$ and have many solutions if $l = \frac{m}{2} = k = 1$. (6)

b) What type of conic section does the quadratic form $4{x_1}^2 + 6x_1x_2 - 4{x_2}^2 = 10$ represents? Express $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ in terms of new coordinates. (8)

OR

12. a) Using Gauss elimination, solve the system (6)

$$x + 2y + z = 3$$
$$2x + 3y + 2z = 5$$
$$3x - 5y + 5z = 2$$
$$3x + 9y - z = 4$$

b) Diagonalize the matrix (8)

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$$

MODULE II

13. a) If $u = f(x-y, y-z, z-x)$,evaluate $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$. (6)

b) Locate all relative extrema and saddle points (if any) of $u(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. (8)

OR

14. a) If $v = x^y$, show that $\frac{\partial^3 v}{\partial x^2 \partial y} = \frac{\partial^3 v}{\partial x \partial y \partial x}$. (6)

b) Find the maxima and minima of $x^3 + y^3 - 3axy$. (8)

MODULE III

15. a) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dydx$ by changing the order of integration. (6)

b) Use triple integral to find the volume of the solid in the first octant bounded by the coordinate planes and the plane $3x + 6y + 4z = 12$. (8)

OR

16. a) Find the area of the plane region enclosed by the given curves $y = \sin x$ and $y = \cos x$ for $0 \le x \le \frac{\pi}{4}$. (6)

b) Using triple integrals find the volume of the cylinder $x^2 + y^2 = 4$ bounded by the planes $z = 0$ and $z = 3$. (8)

MODULE IV

17. a) Test the convergence of the series $\sum_{k=1}^{\infty} \frac{(k+5)!}{5!k!5^k}$. (6)

b) Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{9n^2+3n-2}$. (8)

OR

18. a) Determine whether the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\ k^k}{k!}$ is absolutely convergent or not (6)

b) Examine the convergence of $\frac{1}{4\cdot 6} + \frac{\sqrt{3}}{6\cdot 8} + \frac{\sqrt{5}}{8\cdot 10} + \cdots$ (8)

MODULE V

19. a) Find the Maclaurin series expansion for the function $\cos x$. (6)

b) Find the Fourier sine series for $f(x) = |sinx|$,$-\pi < x < \pi$. (8)

OR

20\. a) Find the half range Fourier sine series representation of $f(x) = x$ in $(0,2)$ (6)

b) Find the Fourier series for $f(x) = \begin{cases} 0, -\pi < x < 0 \\ \pi, 0 < x < \pi \end{cases}$ and deduce that (8)

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$$

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