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SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

SECOND SEMESTER B.TECH DEGREE EXAMINATION (Special), AUGUST 2021

Course Code: **20MAT102**

Course Name: **Vector Calculus, Differential Equations and Transforms**

Max. Marks: **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

		CO
1.	Let $f(x, y) = x^2 e^y$ Find the maximum value of a directional derivative at $(-2,0)$ and the unit vector in the direction in which maximum value occurs.	[1]
2.	If $\vec{r} = xi + yj + zk$ and $\|\vec{r}\| = r$, then prove that the divergence of the vector field $F = \frac{c}{r^3}\vec{r}$ is zero.	[1]
3.	Evaluate $\oint_C \cos x \sin y dx + \sin x \cos y dy$ where C is a triangle with vertices $(0,3), (3,3)$ and $(0,3)$ using Green's theorem.	[2]
4.	Calculate the surface integral $\iint_\sigma xzds$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant.	[2]
5.	Find a basis of the solution of the ODE $(x^2 - x)y'' - xy'' + y = 0$ if $y_1(x) = x$ is one of the solution of given ODE.	[3]
6.	Solve the Euler Cauchy equation $x^2 y'' - 5xy' + 9y = 0$.	[3]
7.	Find the Laplace transform of $f(t)$ where $f(t) = \cos(at + b)$	[4]
8.	Find the inverse Laplace transform of $\frac{se^{-2s}}{s^2 - 1}$	[4]
9.	Find the Fourier transform of $f(x) = \begin{cases} 1 & \|x\| \le 1 \\ 0 & \|x\| > 1 \end{cases}$	[5]
10.	Find the Fourier cosine transform of $f(x) = \begin{cases} k & 0 < x < a \\ 0 & x > a \end{cases}$	[5]

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

			CO	Marks
11.	a)	Prove that the line integral $\int_C y\sin x dx - \cos x dy$ is independent of path and hence evaluate it from $(0,1)$ to $(\pi,-1)$.	[1]	(7)
	b)	Find curl (curl F) and Div (curl F) where $\vec{F} = x^2 yi - 2xzj + 2yzk$	[1]	(7)

OR

			CO	Marks
12.	a)	Find the work done by the force $F = xyi + yzj + zxk$ on a particle that moves along the curve $r(t) = ti + t^2 j + t^3 k$, $0 \le t \le 1$.	[1]	(7)
	b)	Check whether $\bar{F} = 2xy^3 i + (1 + 3x^2y^2)j$ is a conservative field on the entire XY plane. If so find the potential function for it.	[1]	(7)

MODULE II

			CO	Marks
13.	a)	Use divergence theorem find the outward flux of the vector field $F = 2xi + 3yj + z^2k$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$.	[2]	(7)
	b)	Using Greens theorem find the work done by the force $F = (e^{2x} - y^3)i + (\sin y + x^3)j$ on a particle that moves once around a circle $x^2 + y^2 = 1$ in counter clock wise direction.	[2]	(7)

OR

			CO	Marks
14.	a)	Use Stokes theorem to evaluate $\oint_C f.dr$ where $f = 2zi + +3xj + 5yk$, σ is the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \ge 0$.	[2]	(7)
	b)	Find the flux of the vector field $f = (x+y)i + (y+z)j + (x+z)k$ over the surface $\sigma : x + y + z = 2$ in the first octant oriented upwards.	[2]	(7)

MODULE III

			CO	Marks
15.	a)	Solve $\frac{d^2y}{dx^2} + y = \csc x$ by the method of variation of parameters.	[3]	(7)
	b)	Solve $y'' - y = e^x \sin 2x$ by the method of undetermined coefficients.	[3]	(7)

OR

			CO	Marks
16.	a)	Solve the non-homogeneous ODE $y'' - 5y' - 6y = e^{3x} + \sin x$	[3]	(7)
	b)	Solve the homogeneous ODE $\frac{d^4y}{dx^4} + 4y = 0$	[3]	(7)

MODULE IV

			CO	Marks
17.	a)	Solve the differential equation $y'' - 4y' + 3y = e^{-t}, y(0) = y'(0) = 1$ using Laplace transform.	[4]	(7)
	b)	Evaluate the following (i) $L\left\{\int_0^t \frac{e^t \sin t}{t} dt\right\}$ (ii) $L\{t^3 e^{-3t}\}$	[4]	(7)

OR

			CO	Marks
18.	a)	Using convolution theorem find the inverse Laplace transform of the function $F(s) = \frac{s}{(s-1)(s^2+4)}$	[4]	(7)
	b)	Evaluate the following (i) $L\left\{\frac{\sin^2 t}{t}\right\}$ (ii) $L^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$	[4]	(7)

MODULE V

			CO	Marks
19.	a)	Compute the Fourier transform of the function $f(x) = e^{-x^2}$	[5]	(7)
	b)	Obtain the Fourier sine transform of $f(x) = \frac{1}{x}$	[5]	(7)

OR

			CO	Marks
20.	a)	Obtain the Fourier cosine transform of $f(x) = \frac{e^{-ax}}{x}$	[5]	(7)
	b)	Find the Fourier integral representation of $f(x) = \begin{cases} 1 & \|x\| \le 1 \\ 0 & \|x\| > 1 \end{cases}$. Hence evaluate the integral $\int_0^\infty \frac{\sin \omega}{\omega} d\omega$.	[5]	(7)

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