120A5

A Total Pages: **3**

Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING
KOTTAYAM, KERALA

(AN AUTONOMOUS COLLEGE AFFILIATED TO
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER B.TECH DEGREE EXAMINATION(R), MARCH-APRIL 2021

Course Code: **20MAT101**

Course Name: LINEAR ALGEBRA AND CALCULUS

Max. Marks: **100** **Duration:** **3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Determine the value of 'k' such that the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ k & 13 & 10 \end{bmatrix}$ is 2.
2. Reduce the quadratic form into the principal axes form.
$x^2 + 2y^2 + z^2 - 2xy + 2yz$
3. Use chain rule to find $\frac{dw}{dx}$ at (0,1,2) for $w = xy + yz,\ y = sinx\ , z = e^x$
4. Find the slope of the surface $z = 10 - 4x^2 - y^2$ in the x direction and in the y direction at the point (1,2,2).
5. Evaluate $\iint_R (x^2 + y^2)dxdy$ where R is the region taken over the first quadrant for which $x + y \leq 1$.
6. Evaluate $\int_0^1 \int_0^{\ln 3} \int_0^{\ln 2} e^{2x+y-z}\, dz\, dy\, dx$.
7. Express the repeating decimal 5.646464..........as a fraction .
8. Determine whether the series$\sum_{k=2}^{\infty}(-1)^k \frac{k}{k-1}$ converges.
9. Find the Taylor series expansion for the function e^x about $x_0 = -1$.
10. Evaluate the coefficient a_0 in the Fourier series expansion for $f(x) = |sinx|$ in $-\pi < x < \pi$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Test for consistency and solve $2x - y + 3z = 8,\ -x + 2y + z = 4,\ 3x + y - 4z = 0$. (6)

b) Find out what kind of conic section or pair of straight lines is given by the quadratic form $x^2 - 12\,xy + y^2 = 70$ and express $\begin{bmatrix} x \\ y \end{bmatrix}$ in terms of new coordinates. (8)

OR

12. a) Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to Row Echelon form and hence find its rank. (6)

b) Diagonalize the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. (8)

MODULE II

13. a) Let $w = 4x^2 + 4y^2 + z^2$, $x = \rho \sin\varphi \cos\theta$, $y = \sin\varphi \sin\theta$, $z = \rho\cos\varphi$. Find $\frac{\partial w}{\partial \rho}, \frac{\partial w}{\partial \varphi}, \frac{\partial w}{\partial \theta}$. (6)

b) Find the local linear approximation L(x,y) to $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$ at the point P(4,3). Compare the error in approximating f by L at the specified point Q(3.92, 3.01) with the distance between P and Q. (8)

OR

14. a) If $u = f(\frac{x}{y}, \frac{y}{z}, \frac{z}{x})$ find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (6)

b) Locate all relative extrema and saddle points (if any) of $f(x,y) = 4xy - y^4 - x^4$. (8)

MODULE III

15. a) Evaluate $\iint_R y\, dA$ where R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$. (6)

b) Use triple integral to evaluate the volume of the solid bounded by the surface $y = x^2$ and the planes $y + z = 4$ and $z = 0$. (8)

OR

16. a) Evaluate the integral $\int_0^2 \int_{\frac{y}{2}}^1 \cos(x^2)dx\, dy$ by reversing the order of integration. (6)

b) Use polar coordinates to evaluate $\iint_R \frac{1}{1+x^2+y^2} dA$ where R is the sector in the first quadrant bounded by $y = 0, y = x$ and $x^2 + y^2 = 9$. (8)

MODULE IV

17. a) Determine whether the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+7}{k(k+4)}$ is absolutely convergent. (6)

b) Find the sum of the series $\sum_{k=1}^{\infty} \left(\frac{1}{5^k} - \frac{1}{k(k+1)}\right)$ (8)

OR

18. a) Determine whether the series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}}$ converges. (6)

b) Determine whether the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}$ converges absolutely, converges conditionally or diverges. (8)

MODULE V

19. a) Find the Maclaurin series expansion for the function $\sin x$. (6)

b) Find the Fourier series representation of $f(x) = x^2$ in $[-\pi, \pi\,]$ and deduce the value of $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \ldots \ldots$ (8)

OR

20. a) Obtain the Fourier series expansion for the function $f(x) = \begin{cases} 0, & -\pi < x < \pi \\ x^2, & 0 < x < \pi \end{cases}$ (8)

b) Find the half range sine series of $f(x) = e^x$ in $(0,1)$. (6)

****