120A4

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SAINTGITS COLLEGE OF ENGINEERING
KOTTAYAM, KERALA

(AN AUTONOMOUS COLLEGE AFFILIATED TO
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER B.TECH DEGREE EXAMINATION(S), JULY 2021

Course Code: **20MAT101**

Course Name: LINEAR ALGEBRA AND CALCULUS

Max. Marks: **100** **Duration:** **3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$

2. Find the sum and product of the Eigen values of $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 6 \\ 0 & 0 & 6 \end{bmatrix}$ without using its characteristic equation.

3. Show that the function $z = x^2 - y^2 + 2xy$ satisfies Laplace's equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

4. Find the derivative of $z = 3x^2y^3$ with respect to t along the path $x = t^4, y = t^3$ using chain rule.

5. Find the volume under the surface $z = 3x^3 + 3x^2y$ and over the rectangle $R = \{(x, y): -1 \leq x \leq 3, 0 \leq y \leq 2\}$.

6. Change the order of integration in $\int_0^1 \int_x^1 \frac{x}{x^2+y^2} dydx$ and hence evaluate the same.

7. Test the convergence of the series $\sum_{k=1}^{\infty} \frac{99^k}{k!}$.

8. Express the repeating decimal 0.451141414 .... as a fraction.

9. Find the Taylor series for $f(x) = e^{-x}$ about $x = \ln 3$ up to third degree terms.

10. Find the Fourier half range cosine series of $f(x) = e^x$ in $0 < x < 1$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Using Gauss Elimination method find the solution of the system of equations (7)

$$8y + 6z = -4$$
$$-2x + 4y - 6z = 18$$
$$x + y - z = 2$$

b) Find all eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$ (7)

OR

12. a) Find the matrix of transformation that diagonalize the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. Also write the diagonal matrix. (7)

b) What kind of conic section is given by the quadratic form $x^2 - xy + y^2 = 8$ Transform it to principal axis. (7)

MODULE II

13. a) Let $w = \ln(e^r + e^s + e^t + e^u)$. Show that $w_{rstu} = -6e^{r+s+t+u-4w}$. (7)

b) Locate all relative extrema of $f(x, y) = x^2 + xy + y^2 - 6x$. (7)

OR

14. a) Find the Local linear approximation to $f(x, y) = \ln xy$ at the point (1,2). Use it to approximate $f(1.01, 2.01)$. (7)

b) The length and width of a rectangle are measured with errors of at most 3% and 4%, respectively. Use differentials to approximate the maximum percentage error in the calculated area. (7)

MODULE III

15. a) Evaluate $\int\int_R (2x - y^2)dA$ over the triangular region R enclosed between the lines $y = -x + 1, y = x + 1$ and $y = 3$. (7)

b) Evaluate the double integral by converting to polar coordinates (7)

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2 + y^2)dxdy$$

OR

16. a) Find the mass and center of gravity of the lamina with density $\delta(x, y) = x + 2y$ is bounded by the x-axis, the line $x = 1$, and the curve $y = \sqrt{x}$. (7)

b) Use triple integral to find the volume of the solid in the first octant bounded by the coordinate planes and the plane $3x + 6y + 4z = 12$. (7)

MODULE IV

17. a) Determine whether the series converges (i) $\sum_{k=1}^{\infty}\left(\frac{-3}{4}\right)^{k-1}$ (ii) $\sum_{k=1}^{\infty}\frac{1}{(k+3)(k+4)}$ (7)

b) Determine the convergence or divergence of the series $\sum_{k=1}^{\infty}(-1)^k\frac{(2k+1)!}{2^k}$. (7)

OR

18. a) Test whether the following series is absolutely convergent or conditionally Convergent $\sum_{k=3}^{\infty}(-1)^k\frac{\ln k}{k}$. (7)

b) Check the convergence of the series $\frac{3}{4}+\frac{3.4}{4.6}+\frac{3.4.5}{4.6.8}+\frac{3.4.5.6}{4.6.8.10}+\cdots$. (7)

MODULE V

19. a) Obtain the Fourier series for the function $f(x)=(\pi-x)^2$, $\quad -\pi<x<\pi$ (7)

b) Find the half range sine series for $f(x)=x\cos x$ in $(0,\pi)$. (7)

OR

20. a) Find the Fourier series expansion of $f(x)=x^2$ in $(-\pi,\pi)$. Using Parseval's identity deduce that,

$$\frac{\pi^4}{90}=1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots$$ (7)

b) Obtain the Fourier series for the function $f(x)=\pi x$ in $[0,2]$ with period 2. (7)

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