186A3

A Total Pages **2**

Register No: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING
KOTTAYAM, KERALA

(AN AUTONOMOUS COLLEGE AFFILIATED TO
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH. DEGREE EXAMINATION(S), JULY 2021
GEOMECHANICS AND STRUCTURES

Course Code: 20CEGST101

Course Name: APPLIED MATHEMATICS FOR CIVIL ENGINEERS

Max. Marks: **60** **Duration:** **3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Write the generating function for $P_n(x)$ and show that $P_n(1) = 1$.
2. Find the Laplace transform of $f(t) = tcost$.
3. Prove that Kronecker delta is a second order mixed tensor.
4. Form the differential equation corresponding to the integral equation
   $y(x) = \int_0^x t(t-x)y(t)dt + \frac{1}{2}x^2$.
5. Find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $2x^2 - x^3$, using D'Alemberts formula.
6. Solve $q\,(q^2 + s) = pt$ by Monge's method.
7. What is the convergent criteria in Gauss Seidal iterative method?
8. Estimate the value of the integral $\int_0^6 \frac{1}{1+x^2}dx$ using two-point rule.

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. Prove that $\int_{-1}^{1} x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$. (6)

OR

10. Show that $\frac{d}{dx}\left[xJ_n(x)J_{n+1}(x)\right] = x\left[J_n{}^2(x) - J_{n+1}{}^2(x)\right]$. (6)

MODULE II

11. Solve the boundary value problem $\frac{d^2y}{dt^2} + 9y = \cos 2t$ with $y(0) = 1, y\left(\frac{\pi}{2}\right) = -1$ using Laplace transform. (6)

OR

12. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, |x| \le 1 \\ 0 \quad\ , |x| > 1 \end{cases}$. Hence evaluate $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$. (6)

MODULE III

13. If a covariant tensor has components $x + y,\ xz,\ 2z - y^2$ in cartesian co-ordinate system, then find its components in cylindrical co-ordinates. (6)

OR

14. Show that the velocity of a fluid at any point is a contravariant tensor of rank 1. (6)

MODULE IV

15. Convert the differential equation $y'' + y = 0$ with the initial conditions $y(0) = 0, y'(0) = 0$ into an integral equation. (6)

OR

16. Find the solution of the integral equation $Y(x) = \sin x + \lambda \int_0^{2\pi} \cos(x+t) Y(t)\, dt$ by iterative method. (6)

MODULE V

17. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. At $t = 0$, the string is given a shape defined by $F(x) = \mu x(l - x)$, where μ is a constant, and then released. Find the displacement of the string at any distance x from one end at any time t. (6)

OR

18. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the following conditions: (6)

$$u(0,y) = u(l,y) = u(x,0) = 0 \text{ and } u(x,a) = \sin\frac{n\pi x}{l}.$$

MODULE VI

19. Apply factorization method to solve the system of equations (6)
$3x + 2y + 7z = 4;\ 2x + 3y + z = 5;\ 3x + 4y + z = 7$

OR

20. Solve the partial differential equation $\Delta^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y,\ x = 3 = y$ with $u = 0$ on the boundary and mesh length 1 unit. (6)

****