APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M.TECH DEGREE EXAMINATION
Mechanical Engineering
(Machine Design)
04 ME6501-Advanced Engineering Mathematics

Time: 3 hrs | Max. Marks: 60

PART A

(Answer all questions. Each question carry 3 marks).

1. Find the extremals of the functional $\int_{x_0}^{x_1} \frac{y'^2}{x^3} dx$. (3)
2. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre Polynomials?. (3)
3. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x,0) = 3\sin n\pi x, u(0,t) = 0$ and $u(l,t) = 0$,where$0 < x < 1, t > 0$? (3)
4. Classify the equation $U_{xx} + 4U_{xy} + 4U_{yy} - U_x + 2U_y = 0$ (3)
5. Show that the Kronecker delta is a mixed tensor of order two. (3)
6. A covariant tensor has components $x + y, xy, 2z - y^2$ in Cartesian co-ordinate system. Find its components in spherical co-ordinates. (3)
7. Explain the fundamental principles of design of experiments (3)
8. What is Latin square design? Under what conditions can this design be used? (3)

PART B

(Each full question carries 6 marks).

9. Find the curves on which the functional $\int_0^1 [y'^2 + 12xy]dx$ with $y(0) = 0, y(1) = 1$ can be extremised? (6)

OR

10. Show that the functional (6)

$$\int_0^{\frac{\pi}{2}} \left(2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right) dt$$

such that $x(0) = 0, x\left(\frac{\pi}{2}\right) = 1, y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$ is stationary for $x = \sin t$, $y = \sin t$?

11. Solve the series $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$ (6)

OR

12. Solve in the series equaion $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$ (6)

13. A string is stretched and fastened to two points l apart.Motion is started by displacing the string in the form $y = a\sin(\frac{\pi x}{l})$ from which it is released at time $t = 0$.Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a\sin(\frac{\pi x}{l})cos(\frac{\pi ct}{l})$ (6)

OR

14. The ends A and B of a rod 20 cm long have the temperature at 30^0C and 80^0C until steady state prevails.The temperature of the ends are changed to 40^0C and 60^0C respectively.Find the temperature distribution in the rod at time t. (6)

15. Solve by Crank Nicholson method the equation $U_{xx} = 16U_t$, $0 < x < 1, t > 0$ subject to the conditions $U(x,0) = 0,\ U(0,t) = 0$ and $U(1,t) = 100t$ for 1 time step taking $h = \frac{1}{4}$ (6)

OR

16. The transverse displacement u of a point at a distance x from one end and at any time t of a vibrating string satisfies the equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, with the boundary conditions $u = 0$ at $x = 0,\quad t > 0$ and $u = 0$ at $x = 4,\quad t > 0$ and initial conditions $u = x(4-x)$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0$, $0 \leq x \leq 4$. Solve this equation numerically for one half period of vibration,taking h=1, $k = \frac{1}{2}$ (6)

17. Find the components of first and second fundamental tensors in spherical co-ordinates. (6)

OR

18. Prove that (i)the contraction of the tensor A^p_q is an invariant (6)
(ii)The contraction of the outer product of the tensor A^p and B_q is also an invariant

19. The following are the defective pieces produced by four operators working in turn on four different machines (6)

Machine	Operator			
	B1	B2	B3	B4
A1	34	28	33	29
A2	31	24	35	22
A3	27	20	43	72
A4	28	28	29	26

Perform analysis of variance at 0.05 level of significance to ascertain whether variability in production is due to variability in operator's performance or machine's performance

OR

20. A manufacturer of machine parts considering one of the 4 machines currently in the market. The following is the daily output on 5 randomly selected days for each machine: (6)

Machine I	72	56	68	65	60
Machine II	62	70	66	64	78
Machine III	68	72	74	70	66
Machine IV	64	72	68	68	58

Do the machines have an equal output ? Use $\alpha = 0.01$