APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M. TECH DEGREE EXAMINATION

Electronics And Communication Engineering

(Telecommunication)

04 EC6803 - Random Processes and Applications

Time: 3 Hours　　　　　　　　　　　　　　　　Max. Marks : 60

PART A

(Answer All questions. Each question carries 3 marks)

1. State and prove Baye's theorem on inverse probability.
2. If the joint pdf of (X,Y) is given by $f_{xy}(X,Y) = \frac{x+y}{21}$; x=1,2,3 and y=1,2 . Find marginal pdfs?
3. Let X, Y be i.i.d random variables with $f_x(X) = e^{-x}u(x)$. Let Z= max(X,Y). Find $F_z(Z)$?
4. Define power spectral density. Find the power spectral density for the exponential auto - correlation function $R_{xx}(\tau) = \exp(-\alpha|\tau|)$; $-\infty < \tau < \infty$; $\alpha > 0$ is a parameter.
5. If X is a poisson random variable with parameter a>8. Find E[X] ?
6. If X is an arbitrary random variable with mean $\bar{X}$ and finite variance σ^2. Prove that $P[|X-\bar{X}| \geq \delta] \leq \frac{\sigma^2}{\delta^2}$; for any $\delta > 0$.
7. Define sure convergence and almost sure convergence of a random sequence.
8. State Karhunen-Loeve expansion.

PART B

(Each full question carries 6 marks)

9. The pdf of a random variable is given by $f(x) = \begin{cases} ke^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$. Find (i) Value of k (ii) P[0<X<3]　(iii) P[X>0]　(iv) F(X)

OR

10. If X is a normal variate with mean 20 and S.D 5. Find the probability that (i) P[X>23] (ii) P[|X-20|>5]

11. The joint pdf of two random variables is given by $f_{xy}(X,Y) = \frac{1}{2\pi}\exp\left\{\frac{-1}{2}[x^2 + y^2]\right\}$ for $-\infty < X, Y < \infty$. Compute the probability that {X,Y} are restricted to (i) a 2x2 square (ii) a unit circle

OR

12. Determine the pdf of Y= Sin X, where X is a uniform r.v on $(-\pi, \pi)$.

13. Let X be a binomial r.v . Find first and second moment of X

OR

14. Let X~ N(μ, σ^2). Find (i) mgf (ii) $\theta^{(1)}(0)$ and (iii) $\theta^{(2)}(0)$

15. Consider a random process X(t) =A cos (wt +θ), where A and θ are independent and uniform r.v over (-k,k) and $(-\pi, \pi)$ respectively. Find (i) mean of X(t) (ii) Correlation function of X(t) (iii) Covariance function of X(t) and (iv) Variance of X(t) .

OR

16. A random vector X=(X_1, X_2,X_3)^T has covariance matrix $K_x = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Design a non-trivial transformer that will generate from X, a new random vector Y whose components are uncorrelated.

17. Let X be a poisson r.v with parameter a>0. Compute the Chernoff bound for P[X ≥K], where K>a.

OR

18. Consider a random sequence X[n] =$\sum_{m=1}^{n} \alpha^{n-m}$W[m]; n≥ 1, with $|\alpha| < 1$. Let W[n] be a Bernoulli random sequence with W[n] = 1 with probability p and W[n] = 0 with probability q= 1-p. Find mean and variance of X[n] ?

19. Consider the WSS process X(t) = A Cos (2πf_0t + θ); -∞ $< t < \infty$, where A~ N(0,1) and $\theta$is uniformly distributed over [-π,π] and both A and θ are independent. Determine whether (i) X(t) is ergodic in mean (ii) X(t) is ergodic in power and (iii) X(t) is ergodic in correlation

OR

20. Let the WSS random process X(t) be a m.s periodic with period T, where

X(t) =$\sum_{n=-\infty}^{\infty} A_n \, e^{jw_0 \, nt}$ with $w_0 = \frac{2\pi}{T}$ and random fourier coefficients

$A_n = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} X(\tau)\, e^{-jw_0\, n\tau} d\tau$. Show that (i) Mean E[$A_n$] =$\mu_x \delta$[n]

(ii) Correlation E[$A_n A_m^*$] = $\propto_n \delta$[m-n]

(iii)Mean-square value $\alpha_n = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} R_{xx}(\tau)\, e^{-jw_0\, n\tau} d\tau$.