Name:....................
Reg. No:................

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester M.tech Degree Examination
Inter-disciplinary Engineering
(Robotics and Automation Engineering)

04EC6901 - Advanced Mathematics & Optimization Techniques

Time: Three hrs Max. Marks: 60

PART A

(Answer all questions. Each question carry 3 marks).

1. Determine whether the set of nthdegree polynomial on the variable t with real coefficient is a vector space under standard addition and scalar multiplication for polynomial. If the scalars are restricted to being real. (3)

2. Determine whether T from $V \to V$ defined by $T(v) = kv$, $\forall v \in V$ and any scalar k is linear. (3)

3. Define an inner product. (3)

4. Convert the following 0-1 programming problem into standard form: (3)

$$\begin{aligned} \text{Max Z } &= 5x_1 - 9x_2 \\ \text{Sub to } \quad & 2x_1 + x_2 \leq 5 \\ & 34x_1 + 6x_2 \geq 4 \\ & x_1, x_2 \in 0, 1 \end{aligned}$$

5. Distinguish between integer programming problem and linear programming problem. Give examples. (3)

6. Solve the following LPP graphically (3)

$$\begin{aligned} \text{Maximise } \quad & z = 2x_1 + 3x_2 \\ \text{Subject to } \quad & x_1 + x_2 \geq 6 \\ & 7x_1 + x_2 \geq 14 \\ & x_1, x_2 \geq 0 \end{aligned}$$

7. State Kuhn-Tucker conditions for a non linear programming problem having a maximization objective function (3)

8. List and explain the basic assumptions of linear programming problem. (3)

PART B

(Answer all questions)

9. Determine whether the set $B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ is a basis for R^2, considered as a column matrix. (6)

OR

10. Which of the following subsets of R^3 are subspaces? (6)

    1. The plane of vectors (b_1, b_2, b_3) with first component $b_1 = 0$
    2. The vectors (b_1, b_2, b_3) with $b_2.b_3 = 0$

11. A linear transformation $T : R^2 \to R^2$ has the property that $T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \& T \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$. Determine $T(v)$ for any vector $v \in R^2$ (6)

OR

12. Let $G : R^3 \to R^3$ be the linear transformation defined by $G(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find a basis & the dimension of image of G and kernal of G. (6)

13. Apply Gram-Schmidth orthogonalisation process to the basis $B = \{(1, 0, 1), (1, 0, -1), (0, 3, 4)\}$ of the inner product space R^3 to find orthogonal & orthonormal basis of R^3. (6)

OR

14. Construct a singular value decomposition of $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$ (6)

15. Solve using dual simplex method (6)

$$\begin{aligned} \text{Max} \quad z &= -4x_1 - 3x_2 \\ \text{Subject to} \quad x_1 + x_2 &\leq 1 \\ x_2 &\geq 1 \\ -x_1 + 2x_2 &\leq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

OR

16. Find the optimum feasible solution using Big M method. (6)

$$\begin{aligned} \text{Min z} &= 10x_1 + 15x_2 + 20x_3 \\ \text{subject to} \quad 2x_1 + 4x_2 + 6x_3 &\geq 24 \\ 3x_1 + 9x_2 + 6x_3 &\geq 30 \\ x_1, \; x_2, x_3 &\geq 0 \end{aligned}$$

17. Solve the following integer programming problem optimally using branch-and-bound technique. (6)

$$\begin{aligned} \text{Max z} &= 6x_1 + 8x_2 \\ \text{Subject to} \quad 4x_1 + 5x_2 &\leq 22 \\ 5x_1 + 8x_2 &\leq 30 \\ x_1, \; x_2 &\geq 0 \;\; \textit{and integers} \end{aligned}$$

OR

18. Find the optimum integer solution for the following linear programming problem using Gomory's Cutting plane method. (6)

$$\text{Max z} = 5x_1 + 8x_2$$
$$\text{Subject to } \; x_1 + 2x_2 \leq 8$$
$$4x_1 + x_2 \leq 10$$
$$x_1, \; x_2 \geq 0 \; \textit{and integers}$$

19. Solve the following using Kuhn-Tucker conditions (6)

$$\text{Maximise } \; z = 3{x_1}^2 + 14x_1x_2 - 8{x_2}^2$$
$$\text{Subject to } \; 3x_1 + 6x_2 \leq 72$$
$$x_1, x_2 \geq 0$$

OR

20. Solve the non linear programming problem using Lagrangian method (6)

$$\text{Maximise } \; z = 4x_1 - 0.02{x_1}^2 + x_2 - 0.02{x_2}^2$$
$$\text{Subject to } \; x_1 + 2x_2 = 120$$
$$x_1, x_2 \geq 0$$