

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
FIRST SEMESTER M.TECH DEGREE EXAMINATION, FEBRUARY 2016

**Mechanical Engineering**  
**(Machine Design)**

**04ME 6501 Advanced Engineering Mathematics**

Max. Marks: 60

Duration: 3 Hours

**Part A: 3 marks each**

1. Show that the geodesics on a plane are straight lines.
2. Express  $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$  in terms of Legendre polynomials.
3. Using d'Alembert's method find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection  $f(x) = k(\sin x - \sin 2x)$
4. Determine whether the following equation is elliptic or hyperbolic?  
$$(x+1)u_{xx} - 2(x+2)u_{xy} + (x+3)u_{yy} = 0$$
5. Show that any inner product of the tensor  $A_r^p$  and  $B_t^{qs}$  is a tensor of rank 3
6. Show that the velocity of a fluid at any point is a covariant tensor of order one.
7. What is Latin square design? Under what conditions can this design be used?
8. Explain the fundamental principles of design of experiments.

**Part B: 6 marks each**

9. Show that the functional  $\int_0^{\pi/2} (2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2) dt$  such that  $x(0)=0$ ,  $x(\pi/2)=-1$ ,  $y(0)=0$ ,  $y(\pi/2)=1$  is stationary for  $x=-\sin t$ ,  $y=\sin t$ .

OR

10. Find the curves on which the functional  $\int_0^1 [y'^2 + 12xy] dx$  with  $y(0)=0$  and  $y(1)=1$  can be extremised.

11. Prove that 
$$\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = \begin{cases} 0, & \alpha \neq \beta \\ \frac{1}{2} [J_{n+1}(\alpha)]^2, & \alpha = \beta \end{cases}$$

OR

12. Solve in series the equation 
$$9x(1-x) \frac{d^2 y}{dx^2} - 12 \frac{dy}{dx} + 4y = 0$$

13. A tightly stretched string of length  $l$  with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity  $v_0 \sin^3 \frac{\pi x}{l}$ . Find the displacement  $y(x,t)$

OR

14. The ends A and B of a rod 20cm long have the temperature at  $30^\circ\text{C}$  and  $80^\circ\text{C}$  until steady state prevails. The temperature of the ends are changed to  $40^\circ\text{C}$  and  $60^\circ\text{C}$  respectively. Find the temperature distribution in the rod at time  $t$

15. Using Crank Nicholson method solve  $u_{xx} = 16u_t$ ,  $0 \leq x \leq 1$ ,  $t \geq 0$  given

$$u(x,0) = 0, u(0,t) = 0, u(1,t) = 50t. \text{ Compute } u \text{ for 2 steps in } t \text{ direction taking } h=1/4$$

OR

16. Evaluate the pivotal values of the equation  $u_{tt} = 16u_{xx}$ , taking  $h=1$  upto  $t=1.25$ . The boundary conditions are  $u(0,t) = u(5,t) = 0$ ,  $u(x,0) = x^2(5-x)$

17. A covariant tensor has components  $2x - z, x^2 y, yz$  in Cartesian co-ordinate system find its components in spherical co-ordinates.

OR

18. Find the components of first and second fundamental tensors in cylindrical co-ordinates.

19. A professor wishes to select a good text from four different ones available. He has 37 students whom he distributes at random into four groups of 9, 10, 11 and 7 students assigning the books at random to the groups. After the course is over, all the students take the same test obtaining scores as given in table. Explain if any of the four books is to be preferred over the others

Text books	A	B	C	D
Scores obtained in the test	68	41	54	44
	68	47	44	51
	69	54	51	69
	60	65	56	59
	73	32	47	59
	64	73	61	55
	71	44	59	66
	67	48	49	
	75	64	41	
		54	31	
		73		

OR

20. A farmer applies three types of fertilizers on 4 separate plots. The figure on yield per acre are given below

Fertilizers	Yield			
	Plot A	Plot B	Plot C	Plot D
Nitrogen	6	4	8	6
Potash	7	6	6	9
Phosphates	8	5	10	9

Find out if plots are materially different in fertility, as also, if the three fertilizers make any material difference in yields.