APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M.TECH DEGREE EXAMINATION
Inter-disciplinary Engineering
(Robotics and Automation Engineering)
04 EC 6901 Advanced Mathematics & Optimization techniques

Time: 3 hrs — Max. Marks: 60

PART A

(Answer all questions. Each question carry 3 marks).

1. Define linear independence and dependence. (3)
2. Define linear transformation. (3)
3. Define the term orthonormal sets . (3)
4. What are the properties of linear programming solution. (3)
5. With the help of an example define integer programming problem. (3)
6. State Kuhn-Tucker conditions for the following nonlinear programming problem: (3)

$$\begin{aligned} \text{Maximise} \quad & z = 3{x_1}^2 + 14x_1x_2 - 8{x_2}^2 \\ \text{Subject to} \quad & 3x_1 + 6x_2 \leq 72 \\ & x_1, x_2 \geq 0 \end{aligned}$$

7. Mention Branch and bound algorithm applied to maximization problem. (3)
8. What is quadratic programming? (3)

PART B

(Each full question carries 6 marks).

9. Determine whether $\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2\times2} | b = c = 0 \right\}$ is a vector space under standard matrix addition and scalar multiplication (6)

OR

10. Determine the co-ordinate representation of the matrix $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$ with respect to the basis $S = \left\{ \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ (6)

11. Identify the kernal and image of the linear transformation $T : \mathbb{P}^2 \rightarrow \mathbb{M}_{2\times2}$ defined by (6)

$$T(at^2 + bt + c) = \begin{bmatrix} a & 2b \\ 0 & a \end{bmatrix}$$

for all real numbers a and b

OR

12. Find the matrix representation for the linear transformation $T : \mathbb{M}_{2\times2} \to \mathbb{M}_{2\times2}$ defined by (6)

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2b+3c & 2b-3c+4d \\ 3a-4-5d & 0 \end{bmatrix}$$

13. Explain the process of QR decomposition. (6)

OR

14. Construct an ortho-normal set from the following linearly independent sets. (6)

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, x_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

15. Solve using dual simplex method (6)

$$\begin{aligned} \text{Min} \quad z &= 2x_1 + 4x_2 \\ \text{Subject to} \quad 2x_1 + x_2 &\geq 4 \\ x_1 + 2x_2 &\geq 3 \\ 2x_1 + 2x_2 &\leq 12 \\ x_1, x_2 &\geq 0 \end{aligned}$$

OR

16. Solve the following LPP using Big M Method (6)

$$\begin{aligned} \text{Min} \quad z &= 2x_1 + 3x_2 \\ \text{Subject to} \quad x_1 + x_2 &\geq 6 \\ 7x_1 + x_2 &\geq 14 \\ x_1, x_2 &\geq 0 \end{aligned}$$

17. Consider the capital budgeting problem where 5 projects are being considered for execution over the next 3 years. The expected returns for each project and the early expenditure are shown below. Assume that each approved project will be executed over the 3-year period. The objective is to select a combination of projects that will maximise the total returns (6)

	Expenditure for			
Project	Year 1	Year 2	Year 3	Returns
1	5	1	8	20
2	4	7	10	40
3	3	9	2	20
4	7	4	1	15
5	8	6	10	30
Max: funds	25	25	25	-

Formulate the problem as a zero - one integer programming problem and solve it by the additive algorithm.

OR

18. Find the optimum integer solution of the following programming problem. (6)

$$\begin{aligned} \text{Max} \quad z &= 5x_1 + 8x_2 \\ \text{Subject to} \quad x_1 + x_2 &\leq 8 \\ 4x_1 + x_2 &\leq 10 \\ x_1, x_2 \geq 0 \quad &\text{and integer} \end{aligned}$$

19. Solve using Lagrangian method (6)

$$\begin{aligned} \text{Maximise} \quad z &= {x_1}^2 + 2{x_2}^2 + {x_3}^2 \\ \text{Subject to} \quad 2x_1 + x_2 + 2x_3 &= 30 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

OR

20. Solve the following using Kuhn-Tucker conditions (6)

$$\begin{aligned} \text{Maximise} \quad z &= x_1^2 + x_1x_2 - 2x_2^2 \\ \text{Subject to} \quad 4x_1 + 2x_2 &\leq 24 \\ x_1, x_2 &\geq 0 \end{aligned}$$