

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M.TECH DEGREE EXAMINATION

Mechanical Engineering
(Machine Design)

04 ME 6501 -Advanced Engineering Mathematics

Max. Marks : 60

Duration: 3 Hours

PART A

Answer All Questions

Each question carries 3 marks

1. Prove that the shortest distance between two points in a plane is a straight line.
2. Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$
3. Using D'Alembert's method find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x)=k(\sin x-\sin 2x)$
4. Classify the equation:

$$3u_{xx} + 4u_{xy} + 6u_{yy} - 2u_x + u_y - u = 0$$

5. Show that the velocity of a fluid at any point is a contravariant tensor of rank one
6. Define covariant and contravariant tensor of order one
7. Explain the fundamental principles of design of experiments
8. What is Latin square design?

PART B

Each question carries 6 marks

9. Find the plane curve of fixed perimeter and maximum area

OR

10. Show that the curve which extremizes the functional $I = \int_0^{\pi/4} ((y'')^2 - y^2 + x^2) dx$ under the conditions $y(0)=0$, $y'(0)=1$, $y(\pi/4) = y'(\pi/4) = 1/\sqrt{2}$ is $y = \sin x$

11. Derive Rodrigue's formula

OR

12. Prove that $J_{1/2}(x) = \sqrt{2/\pi x} \sin x$ and $J_{-1/2}(x) = \sqrt{2/\pi x} \cos x$

13. Derive two dimensional heat equation

OR

14. A tightly stretched string with fixed ends $x=0$ and $x=a$ is initially in a position given by $y=y_0 \sin^3(\pi x/a)$. If it is released from rest from this position, find the displacement $y(x,t)$

15. Evaluate the pivotal values of the equation $u_{tt} = 16 u_{xx}$, taking $h=1$ up to $t=1.25$. The boundary conditions are $u(0,t) = u(5,t) = 0$, $u(x,0) = x^2(5-x)$

OR

16. Using Crank Nicholson method solve $u_{xx} = 16u_t$, $0 \leq x \leq 1$, $t \geq 0$ given $u(x,0) = 0$, $u(0,t) = 0$, $u(1,t) = 100t$ for one steps in t direction taking $h=1/4$

17. A covariant tensor has components $xy, 2y-z^2, xz$ in rectangular co-ordinates find its covariant components in spherical co-ordinates

OR

18. Find the components of metric tensors and conjugate tensor in cylindrical co-ordinates.

19. A trucking company wishes to test the average life of each of the four brands of tyres. The company uses all brands on randomly selected trucks. The records showing the lives (thousands of miles) of tyres are as given in the following table. Test the hypothesis that the average life for each brand of tyres is the same. Assume $\alpha = 0.01$

Brand 1	Brand 2	Brand 3	Brand 4
20	19	21	15
23	15	19	17
18	17	20	16
17	20	17	18
	16	16	

OR

20. For the following data representing the number of units of production per day turned out by five workers using four machines set up the ANOVA table (assumed origin at 20)

WORKER	MACHINE TYPE			
	A	B	C	D
1	4	-2	7	-4
11	6	0	12	3
111	-6	-4	4	-8
1V	3	-2	6	-7
V	-2	2	9	-1