

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M. TECH DEGREE EXAMINATION
Electronics & Communication Engineering
(Telecommunication Engineering)
04EC6801—Applied Linear Algebra

Max. Marks : 60

Duration: 3 Hours

PART A

Answer All Questions

Each question carries 3 marks

1. Given $v_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, Does $\{v_1, v_2\}$ spans \mathbb{R}^2 ?
2. Explain null space, range and nullity.
3. Find the matrix representation of the linear map $G: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $G(x, y) = (2x-3y, 4x+y)$ relative to the basis $\{u_1 = (1, -2), u_2 = (2, -5)\}$ of \mathbb{R}^2 .
4. Verify the Pythagoras Theorem for the following orthogonal set in \mathbb{R}^4 , $u = (1, 2, -3, 4)$, $v = (3, 4, 1, -2)$, $w = (3, -2, 1, 1)$.
5. State Spectral Theorem with example.
6. Determine the eigen values of $A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \end{bmatrix}$ and their algebraic multiplicity.
7. Explain nullspace and nullity of AA^T and $A^T A$.
8. Explain the geometry of pseudo inverse.

PART B

Each question carries 6 marks

9. Find the dimension and basis for row space of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix}$.

OR

10. a) Show that the vector $\begin{bmatrix} 20 \\ 4 \end{bmatrix}$ belongs to span of $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$

b) Prove that the vectors $(1, -1, 1)$, $(0, 1, 2)$ and $(3, 0, -1)$ form a basis for \mathbb{R}^3 .

11. Find the least square solution to the matrix equation $\begin{bmatrix} 2 & -2 \\ -2 & 2 \\ 5 & 3 \end{bmatrix} X = \begin{bmatrix} -1 \\ 7 \\ -26 \end{bmatrix}$ using pseudo inverse method.

OR

12. Find the values of ' λ ' for which the system of equations $x+y+z = 1$, $x+2y+4z = \lambda$, $x+4y+10z = \lambda^2$ will be consistent and also show that for each value of λ the system has a one parameter family of solutions and find these solutions.

13. Find the four fundamental sub spaces of the given matrix $A = \begin{bmatrix} 1 & -2 & -1 & 3 & 2 \\ 2 & -2 & -3 & 6 & 1 \\ -1 & -4 & 4 & -3 & 7 \end{bmatrix}$.

OR

14. Given bases of \mathbb{R}^2 : $S_1 = (u_1 = (1, -2), u_2 = (3, -4))$ and $S_2 = (v_1 = (1, 3), v_2 = (3, 8))$. Find the change of basis matrix from S_1 to S_2 and find the change of basis matrix from S_2 back to S_1 .

15. Find the orthonormal basis of the subspace W of \mathbb{R}^5 spanned by $v_1 = (1, 1, 1, 0, 1)$, $v_2 = (1, 0, 0, -1, 1)$, $v_3 = (3, 1, 1, -2, 3)$, $v_4 = (0, 2, 1, 1, -1)$

OR

16. Find a basis for W^\perp where $W = \text{span} \left\{ \begin{bmatrix} 1 \\ -5 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

17. Compute eigen vectors of the matrix $A = \begin{bmatrix} 4 & 6 & 10 \\ 3 & 10 & 13 \\ -2 & -6 & -8 \end{bmatrix}$.

OR

18. Diagonalize the given matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$.

19. Find SVD of the matrix $A = \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$.

OR

20. Find the pseudo inverse of the matrix $A = \begin{bmatrix} 2 & -2 \\ -1 & 2 \\ 5 & 3 \end{bmatrix}$ using SVD.