Reg No.:_______________ Name:____________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.C.A. DEGREE EXAMINATION, DECEMBER 2018

Course Code: RLMCA103

Course Name: DISCRETE MATHEMATICS

Max. Marks: 60 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1 Show that $(A \cap B)' = A' \cup B'$. (3)

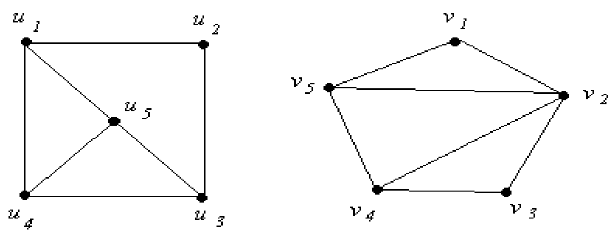
2 Use Euclidean algorithm to obtain x and y satisfying $\gcd(272,\ 1479) = 272x + 1479\,y$. (3)

3 Find the number of arrangements of the letters in TALLAHASSEE. How many of these arrangements have no adjacent A's ? (3)

4 Solve the recurrence relation $6a_n - 7a_{n-1} = 0, n \geq 1, a_3 = 343$. (3)

5 Define Hamilton cycle and Euler circuit with examples. (3)

6 Define isomorphism of graphs. Check whether the graphs given below are isomorphic. (3)



7 What are the contra positive, the converse, and the inverse of the conditional statement "The home team wins whenever it is raining"? (3)

8 Show that the implication $\urcorner(p \to q)$ and $p \wedge \urcorner\, q$ are logically equivalent. (3)

PART B

Answer six questions, one full question from each module and carries 6 marks.

Module I

9 (a) Define an equivalence relation. Let R be a relation defined on a set of positive integers such that for $x, y \in Z^+$, xRy iff $|x - y| < 5$. Check whether R is an equivalence relation? (3)

(b) Let the relation R be defined on the set $A = \{1,2,3,4\}$ as $R = \{(1,2), (2,3), (2,4)\}$. Compute the reflexive, symmetric and transitive closures of R. (3)

OR

10 (a) Let f, g: R → R be defined by $f(x) = x + 1, g(x) = 2x^2 + 3$. Find fog and gof. Is fog = gof ? (3)

(b) Let $f: R - \{2\} \to R - \{1\}$ defined by $f(x) = \frac{x-1}{x-2}$. Show that f is one-to-one and onto. Also find a formula for f^{-1}. (3)

Module II

11 (a) Solve the linear Diophantine equation $24x + 138y = 18$. (4)

(b) Show that 641 is prime. (2)

OR

12 (a) Solve the set of simultaneous congruences (4)

$x \equiv 2(mod\ 3)$; $x \equiv 3(mod\ 5)$; $x \equiv 2(mod\ 7)$.

(b) Show that 1! +2! +3! + …… + 100! gives the remainder 9, when divided by 12. (2)

Module III

13 (a) Find the coefficient of $w^2x^2y^2z^2$ in the expansion of $(2w - x + 3y + z - 2)^{12}$. (2)

(b) A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carried out if (i) there are no restrictions? (ii) there must be an even number of women? (iii) there must be more women than men? (4)

(iv) there must be at least eight men?

OR

14 (a) Determine all integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 7$ where $x_i \geq 0$ for all $1 \leq i \leq 4$. (2)

(b) Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. (4)

Module IV

15 (a) Solve the recurrence relation $2a_n = 7a_{n-1} - 3a_{n-2}, n \geq 2,\ a_0 = 2, a_1 = 5$. (3)

(b) Solve the recurrence relation $a_n - 3a_{n-1} = 5(7^n),\ n \geq 1, a_0 = 2$. (3)

OR

16 Solve the recurrence relation $a_{n+2} - 8a_{n+1} + 16a_n = 8(5^n) + 6(4^n)$ where $n \geq 0, a_0 = 12, a_1 = 5$. (6)

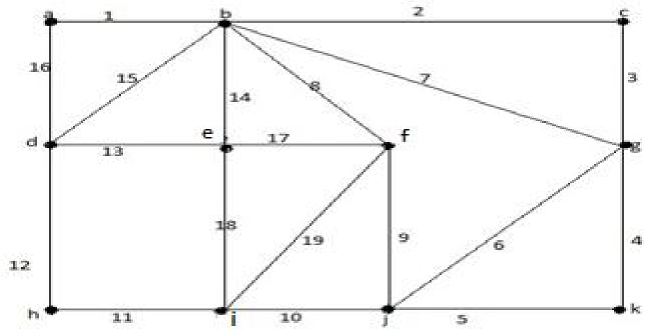
Module V

17 (a) Show that $K_{3,3}$ is non-planar. Define planar graph. State Kuratowski's theorem. (3)

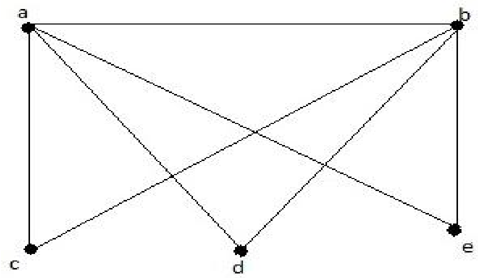
(b) A connected planar graph G has nine vertices having degrees 2, 2, 2, 3, 3, 3, 4, (3) 4, 5. Find the number of edges and the number of faces (or regions) of G.

OR

18 (a) Use Fleury's algorithm to find an Euler circuit in the given graph. (4)



(b) Find the adjacency matrix associated with the graph. (2)



Module VI

19 (a) Show that $p \wedge (\neg q \vee r)$ and $p \vee (q \wedge \neg r)$ are logically not equivalent. (3)

(b) Show that the following argument is valid. Rita is baking a cake. If Rita is baking a cake then she is not practicing her flute. If Rita is not practicing her flute, then her father will not buy her a car. Therefore Rita's father will not buy her a car. (3)

OR

20 (a) Consider these statements of which the first two are premises and third is a valid conclusion. "All lions are fierce". "Some lions do not drink coffee." "Some fierce creatures do not drink coffee". Let $P(x), Q(x)$ and $R(x)$ be the statements "x is a lion" , " x is fierce" and " x drink coffee" respectively. Assuming that the domain consists of all creatures express the statements in the argument using quantifiers and $P(x), Q(x)$ and $R(x)$. (3)

(b) Show that $\neg \exists\ x\, Q(x) \equiv \forall x\ \neg Q(x)$. (3)

****