



G1073

Pages 3

## APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

### Scheme for Valuation/Answer Key

*Scheme of evaluation (marks in brackets) and answers of problems/key*

### SEVENTH SEMESTER B.TECH DEGREE EXAMINATION (S), MAY 2019

Course Code: AE407

#### Course Name: -DIGITAL CONTROL SYSTEM

Max. Marks: 100

Duration: 3 Hours

#### PART A

*Answer any two full questions, each carries 15 marks.*

Marks

- |   |    |  |      |
|---|----|--|------|
| 1 | a) | Merits (any three) – 1.5 marks, Demerits (any three) -- 1.5 marks                | (3)  |
|   | b) | Figure (input and output S/H signal) – 1mark, Each characteristic – 2 marks each | (7)  |
|   | c) | Figure (impulse response) – 2 marks, Derivation – 3 marks                        | (5)  |
| 2 |    | Diagrams – 2 marks each.   | (15) |
|   |    | Explanation – 2 marks each   |      |
|   |    | Derivation – Zero order hold – 3 marks   |      |
|   |    | First order hold – 4 marks   |      |
| 3 | a) | Partial fraction expansion – 3 marks   | (6)  |
|   |    | Inverse z transform: $x(k) = 9k(2^{k-1}) - 2^k + 3$ ; -- 3 marks                 |      |
|   | b) | Each mapping – 3 marks   | (9)  |

#### PART B

*Answer any two full questions, each carries 15 marks.*

- 4 a) The input-output relations can be formulated as (10)

$$\begin{aligned} E(s) &= R(s) - G(s)H(s)E^*(s) \\ C(s) &= G(s)E^*(s) \end{aligned} \quad \text{----- 2 marks}$$

Taking pulse transform on both sides

$$E^*(s) = R^*(s) - GH^*(s)E^*(s)$$

Taking pulse transformation on both sides of

$$\begin{aligned}
 C^*(s) &= [G(s)E^*(s)]^* \\
 &= G^*(s)E^*(s) \\
 &= \frac{G^*(s)R^*(s)}{1 + GH^*(s)} \\
 \therefore \frac{C^*(s)}{R^*(s)} &= \frac{G^*(s)}{1 + GH^*(s)} \\
 \Rightarrow \frac{C(z)}{R(z)} &= \frac{G(z)}{1 + GH(z)}
 \end{aligned}$$

where  $GH(z) = Z[G(s)H(s)]$ .

-----8 marks

b) Derivation (5)

5

(15)

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + G(z)}$$

$$T=1.$$

$$G(z) = Z\left[\left(\frac{1-e^{-sT}}{s}\right)^2 \cdot \frac{1+sT}{s^2} \cdot \frac{1}{s(s+D)}\right]$$

5 marks

$$= \left[1 - z^{-1}\right]^2 \left[Z\left(\frac{1+sT}{Ts^2} \cdot \frac{1}{s(s+D)}\right)\right]_{T=1}$$

$$= \left[1 - z^{-1}\right]^2 \left[Z\left(\frac{1+s}{s^2} \cdot \frac{1}{s(s+D)}\right)\right]$$

$$= \left[1 - z^{-1}\right]^2 \left[Z\left(\frac{1}{s^3}\right)\right]_{T=1}$$

$$= \left(\frac{z-1}{z}\right)^2 \cdot \left[Z\left(\frac{T^2 \cdot z(z+D)}{(z-1)^3}\right)\right]_{T=1} \frac{s+D}{s} = \frac{s+D}{s+D+1}$$

5 marks

$$= \frac{1+Dz}{z} \cdot \frac{z+1}{z-1} = \frac{1+z}{z-1} \cdot \frac{z+1}{z(z-D)}$$

$$\frac{C(z)}{R(z)} =$$

$$= \frac{z+1}{z(z-D)} = \frac{z+1}{z(z-1)+z+1} = \frac{z+1}{z^2+z+2+1} = \frac{z+1}{z^2+2z+1}$$

5 marks



**G1073**

**Pages 3**

- |   |      |
|---|------|
| 6 a) Gain margin & Phase margin explanation                 | (5)  |
| b) Derivation for the velocity error constant ---- 5 marks  | (10) |
| Derivation for the acceleration error constant ---- 5 marks |      |

### PART C

*Answer any two full questions, each carries 20 marks.*

- |  |      |
|--|------|
| 7 a) Splitting the transfer function into standard nested programming equations ---- 5 | (10) |
| marks  |      |

Framing state equation and output equation (nested programming method or observable canonical form)  $x(k+1) = Gx(k) + Hu(k)$ ;  $G = \begin{bmatrix} 0 & -3 \\ 1 & -4 \end{bmatrix}$ ;  $H = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ ;  
 $C = [0 \ 1]$ ;  $D = [0]$ ; --5 marks

- |                           |      |
|---------------------------|------|
| b) Definition --- 2 marks | (10) |
| Explanation ---3 marks    |      |
| Proof – 5 marks           |      |

- |   |      |
|---|------|
| 8 a) Control Law equation $u(k) = -kx(k)$ ; ----2 marks | (10) |
|---|------|

Design Procedure ----8 marks

- |  |      |
|--|------|
| b) State equation and output equation – 3 marks            | (10) |
| z-transform of state equation and output equation – 3 mark |      |
| State transition matrix derivation ---4 mark               |      |

- |  |     |
|--|-----|
| 9 a) i. Controllability Test matrix - $\begin{bmatrix} 2 & -2 \\ 3 & -6 \end{bmatrix}$ , rank=2; State controllable ;--4 | (8) |
| marks  |     |

- |   |  |
|---|--|
| ii. Controllability Test matrix - $\begin{bmatrix} 2 & -2 \\ 0 & 0 \end{bmatrix}$ , rank=1; Not state controllable; --4 |  |
| marks   |  |

- |                          |      |
|--------------------------|------|
| b) Explanation (8 marks) | (12) |
|--------------------------|------|

Deriving the expression (4 marks)

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