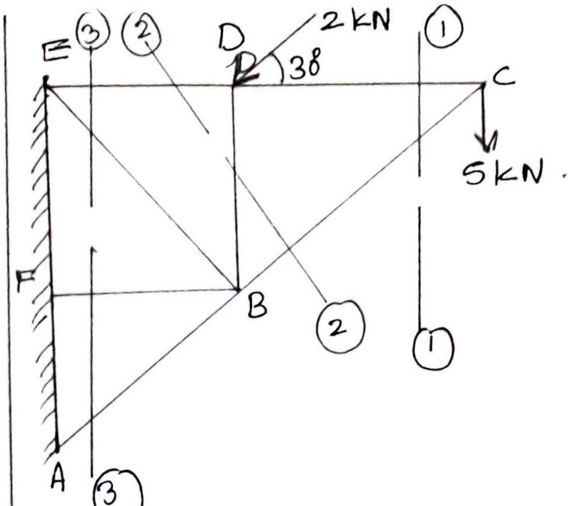
①

1.b



$F_{CB} = 7.07$ kN (C)

$F_{CD} = 5$ kN (T)

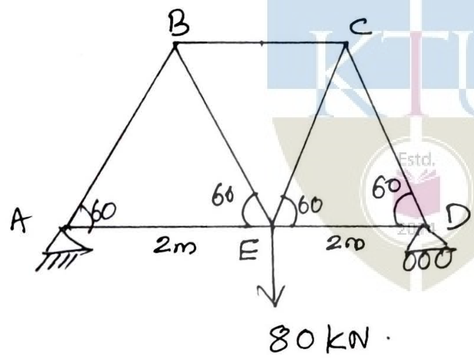
$F_{DE} = 3.27$ kN (T)

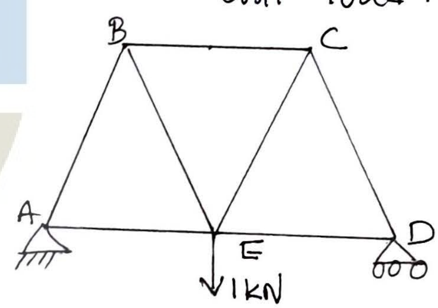
$F_{DB} = 1$ kN (C)

Since the given truss is not a perfect truss, solution of equations resulting from the 3rd section is complicated.
Hence full credit can be given if the student attempted to solve the equations in the 3rd section.

3.b.

Unit load Method.

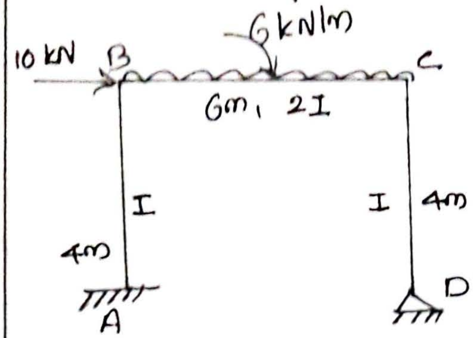




Member	F (kN)	k (kN)	$\frac{L}{AE}$	$\frac{F.k.L}{AE}$
AB	$\bar{4}6.19$	$\bar{0}.58$	0.01	0.267
AE	+23.09	+0.29	"	0.067
BC	$\bar{4}6.19$	$\bar{0}.58$	"	0.267
BE	+46.19	+0.58	"	0.267
CE	+46.19	+0.58	"	0.267
CD	$\bar{4}6.19$	$\bar{0}.58$	"	0.267
ED	+23.09	+0.29	"	0.067

$$\Sigma \frac{FkL}{AE} = 1.47 \text{ mm.}$$

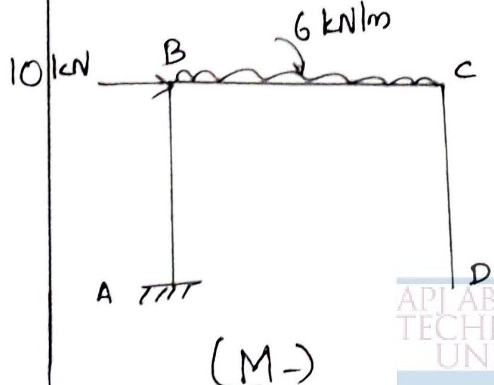
4b. Consistent Deformation Method.

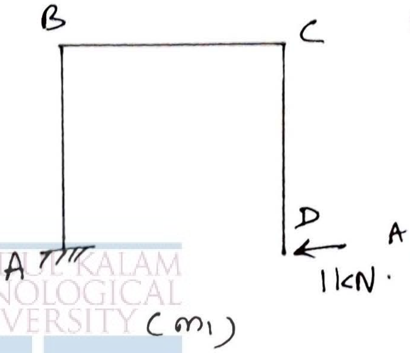


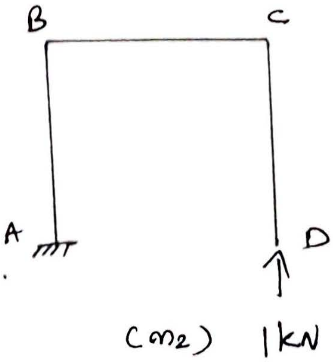
DOR = 2.

Redundants $R_1 = H_D$

$R_2 = V_D$.







Compatibility Equations:

$D_1 = D_{1L} + f_{11} R_1 + f_{12} R_2$

$D_2 = D_{2L} + f_{21} R_1 + f_{22} R_2$

$$\Rightarrow \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} D_{1L} \\ D_{2L} \end{Bmatrix} + \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix}$$

Select any suitable method to determine the deflection terms.

By unit load method:

Portion	DC	CB	BA
origin	D	C	B
Limit	0-4	0-6	0-4
EI	EI	2EI	EI
M	0	$-3x^2$	$\overline{1}08-10x$
m_1	$-x$	-4	-4
m_2	0	x	6

$$D_{11} = \int_0^L M \cdot m_1 \frac{dx}{EI} = \int_0^4 0 + \int_0^6 -3x^2 \times -4 \cdot \frac{dx}{2EI} + \int_0^4 (-108-10x)\,\bar{4} \cdot \frac{dx}{EI}$$

$$= 2480 / EI$$

$$D_{21} = \int_0^L M \cdot m_2 \cdot \frac{dx}{EI} = 0 + \int_0^6 -3x^2 \times x \cdot \frac{dx}{2EI} + \int_0^4 (-108-10x) \times 6 \cdot \frac{dx}{EI}$$

$$= -\frac{3558}{EI}$$

$$f_{11} = \int_0^L m_1\, m_1 \cdot \frac{dx}{EI} = \int_0^4 x^2 \frac{dx}{EI} + \int_0^6 16 \cdot \frac{dx}{2EI} + \int_0^4 16 \cdot \frac{dx}{EI}$$

$$= \frac{400}{3EI}$$

$$f_{22} = \int_0^4 m_2\, m_2 \frac{dx}{EI} = \int_0^4 0 + \int_0^6 x^2 \cdot \frac{dx}{2EI} + \int_0^4 36 \cdot \frac{dx}{EI}$$

$$= 180 / EI$$

$$f_{12} = \int_0^L m_1\, m_2 \frac{dx}{EI} = 0 + \int_0^6 -4x \cdot \frac{dx}{2EI} + \int_0^4 \bar{24} \frac{dx}{EI}$$

$$= -\frac{132}{EI}$$

On solving:

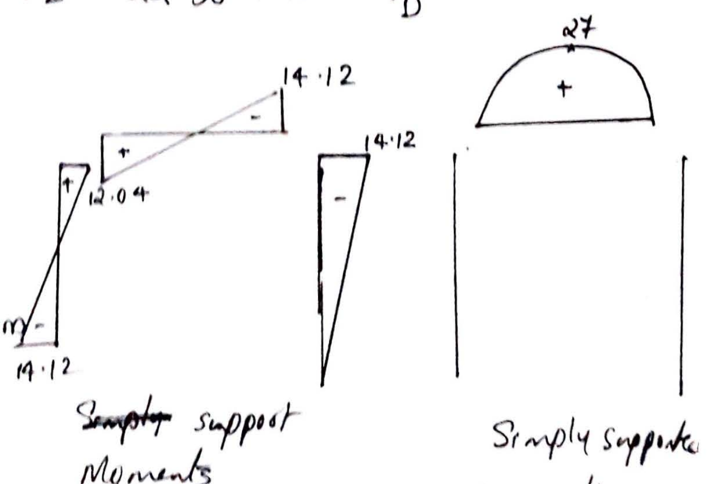
$R_1 = 3.53$ kN $= H_D$; $R_2 = 22.36$ kN $= V_D$

Moments

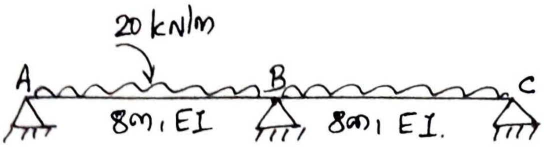
$M_A = -13.84$ kNm

$M_B = 12.04$ kNm

$M_C = -14.12$ kN·m



5a. Strain Energy Method.



DOR = 1

Applying Principle of minimum strain Energy

Let $R_B = R_1$, redundant force

Then $\frac{\partial U}{\partial R_1} = 0$; where U- total SE in the beam

$$\Rightarrow \quad 0 = \int_0^L M \cdot \frac{\partial M}{\partial R_1} \cdot \frac{dx}{EI} \quad \text{—— I}$$

$\Sigma F_v = 0$

$\Rightarrow R_A + R_B + R_C = 20 \times 16 = 320$

By taking moments about A and C, we get

$$R_A = 160 - 0.5 R_B = 160 - 0.5 R_1$$

$$R_C = R_A = 160 - 0.5 R_B = 160 - 0.5 R_1$$

Region	AB	CB
Origin	A	C
Limit (m)	0 − 8	0 − 8
Moment, M	$+R_A x - 20 \cdot x^2/2$	$+R_C \cdot x - 20 \cdot x^2/2$
$\frac{\partial M}{\partial R_1}$	$-0.5x$	$-0.5x$

$$\int_0^4 M \cdot \frac{\partial M}{\partial R_1} \cdot \frac{dx}{EI} = 0$$

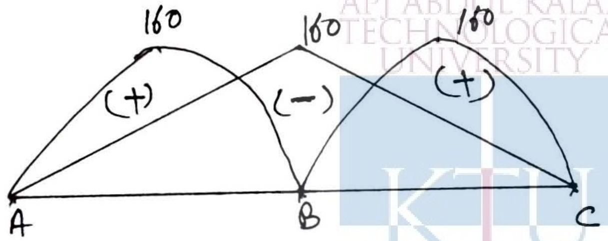
$$\Rightarrow \quad 2 \int_0^8 \left(160x - 0.5xR_1 - 20 \cdot \frac{x^2}{2}\right) \times 0.5 \cdot \frac{dx}{EI} = 0$$

$$\Rightarrow \quad R_1 = 200 \text{ kN} = R_B$$

$$R_A = 60 \text{ kN}$$

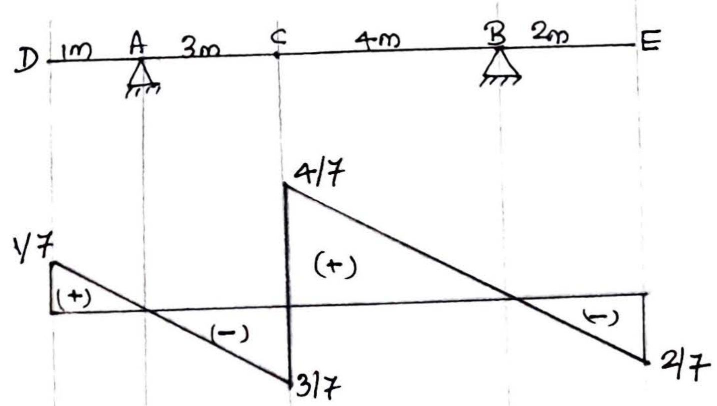
$$R_C = 60 \text{ kN}.$$

$M_A = 0$; $M_B = -160$ kNm. $M_C = 0$.

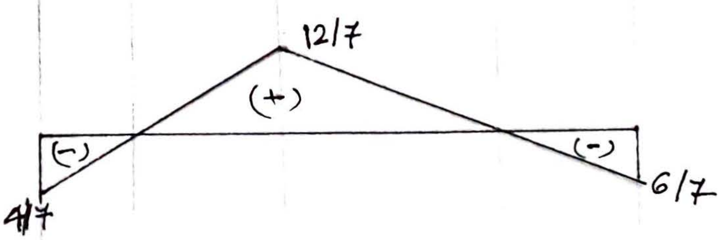


BMD.

5.b.

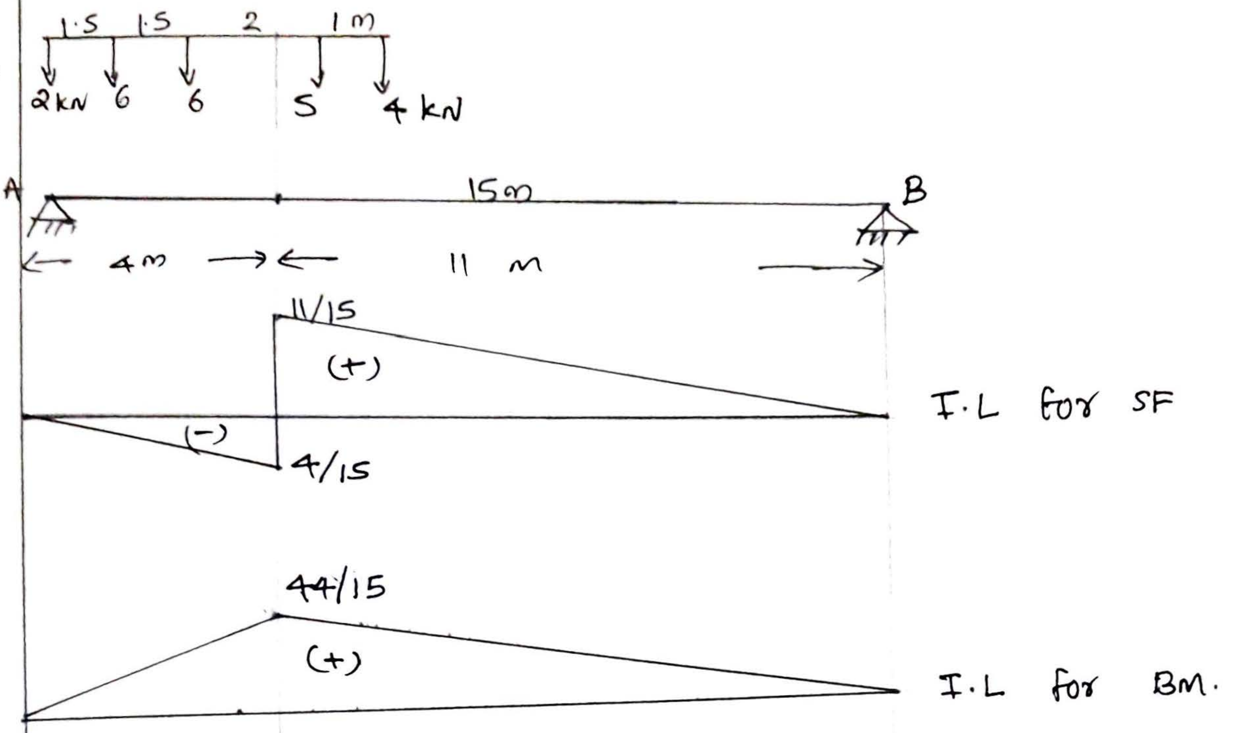


I.L. for S.F @ C



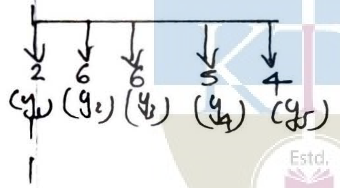
I.L. for B.M @ C

6b.



Maximum +ve SF

Trial 1: 2 kN @ section



$y_1 = {}^{+}0.73$ $\quad y_3 = {}^{+}0.53$

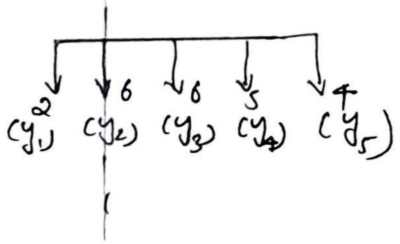
$y_2 = {}^{+}0.63$ $\quad y_4 = {}^{+}0.4$

$y_5 = {}^{+}0.33$

$SF = 2 \times 0.73 + 6 \times 0.63 + 6 \times 0.53 + 5 \times 0.4 + 4 \times 0.33$

$= 11.74$ kN

Trial 2: 6 kN @ section



$y_1 = {}^{-}0.17$ $\quad y_4 = +0.5$

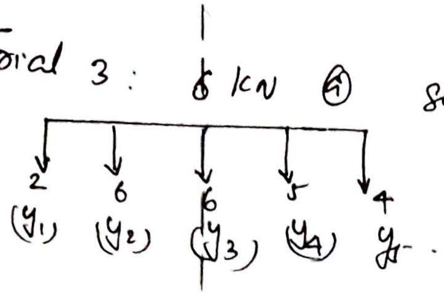
$y_2 = {}^{+}0.73$ $\quad y_5 = 0.43$

$y_3 = {}^{+}0.63$

$SF = 2 \times {}^{-}0.17 + 6 \times 0.73 + 6 \times 0.63 + 5 \times 0.5 + 4 \times 0.43$

$= + 12.04$ kN.

Trial 3 : 6 kN @ section



$y_1 = -0.07$ $\quad y_3 = +0.733$

$y_2 = -0.17$ $\quad y_4 = +0.6$

$y_5 = +0.53$

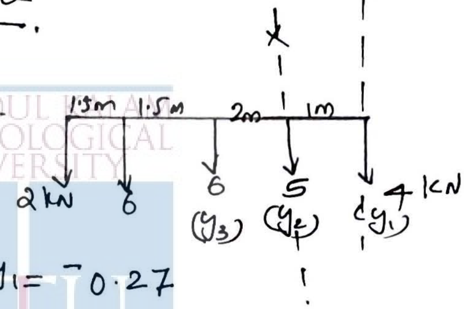
$$+SF = 2\times -0.07 + 6\times -0.17 + 6\times 0.73 + 5\times 0.6 + 4\times 0.53$$

$$= +8.34 \text{ kN}$$

Max +ve SF = 12.04 kN.

Maximum -ve Shear Force

Trial 1 : 4 kN @ section



$$-ve\ SF = 4\times -0.27 + 5\times -0.2 + 6\times -0.07$$

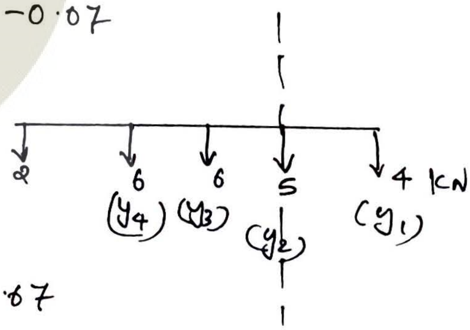
$$= -2.47 \text{ kN}.$$

$y_1 = -0.27$

$y_2 = -0.2$

$y_3 = -0.07$

Trial 2 : 5 kN @ section



$$-ve\ SF = 4\times 0.67 + 5\times -0.27 + 6\times -0.133 + 6\times -0.033$$

$$= 0.334 \text{ kN}$$

$y_1 = +0.67$

$y_2 = -0.27$

$y_3 = -0.133$

$y_4 = -0.033$

Max. -ve SF = -2.47 kN

Maximum Bending Moment at the section

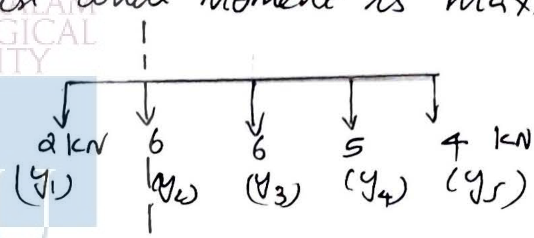
Load @ section	Avg. load on left side.	Avg. load on right side
Trial 1: 5 kN @ section	$\frac{6+6+2}{4} = 3.5$	$\frac{5+4}{11} = 0.8$
Trial 2: 6 kN @ section	$\frac{6+2}{4} = 2$	$\frac{6+5+4}{11} = 1.36$
Trial 3: ✓ 6 kN @ section	$\frac{2}{4} = 0.5$	$\frac{6+6+5+4}{11} = 1.9$

Trial 3 is the required case where moment is maximum.

$y_1 = 2.57$ $\quad y_4 = 2$

$y_2 = 2.93$ $\quad y_5 = 1.73$

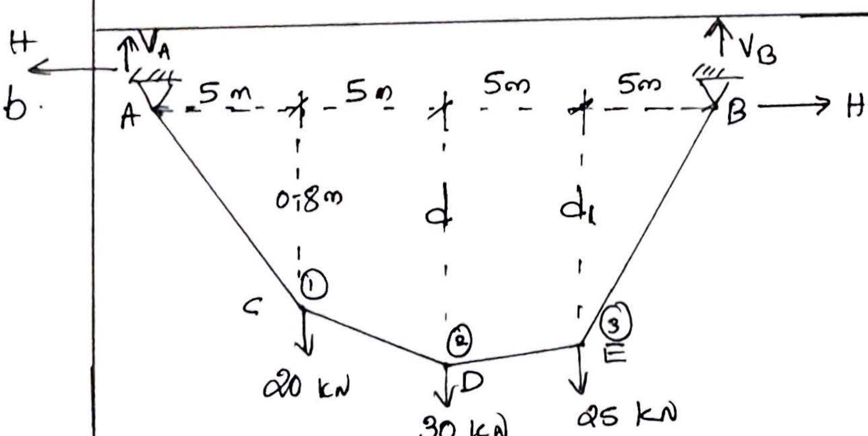
$y_3 = 2.53$



$$\text{Max. B.M} = 2\times2.57 + 6\times2.93 + 6\times2.53 + 5\times2 + 4\times1.73$$

$$= 54.82 \text{ kN-m.}$$

7b.



$\Sigma F_V = 0 \Rightarrow V_A + V_B = 20+30+25$

$\Sigma M_A = 0 \Rightarrow V_A = 36.25 \text{ kN}; \quad V_B = 38.75 \text{ kN}.$

Bending moment, $M_C = 0$

$\Rightarrow 36.25 \times 5 - H \times 0.8 = 0$

$\Rightarrow H = 226.56$ kN

BM, $M_D = 0$

$\Rightarrow 36.25 \times 10 - 226.56 \times d - 20 \times 5 = 0$

$d = 1.158 = 1.16$ m

BM, $M_E = 0$

$\Rightarrow 38.75 \times 5 - 226.56 \times d_1 = 0$

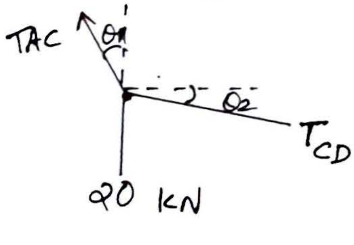
$d_1 = 0.855 = 0.86$ m

Determination of tension values

$T_{AC} = \sqrt{V_A^2 + H^2} = \sqrt{36.25^2 + 226.56^2} = 229.44$ kN.

$T_{BE} = \sqrt{V_B^2 + H^2} = \sqrt{38.75^2 + 226.56^2} = 229.85$ kN

Consider joint C (Joint 1.)



$\tan\theta_1 = \frac{5}{0.8}$

$\theta_1 = 80.91^\circ$

$\tan\theta_2 = 0.36/5$

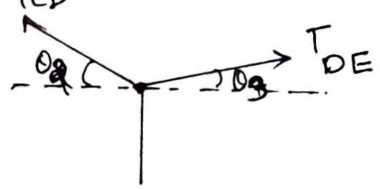
$\theta_2 = 4.12^\circ$

$T_{AC} \sin\theta_1 = T_{CD} \cos\theta_2$

$T_{CD} = \frac{T_{AC} \sin\theta_1}{\cos\theta_2} = 229.44 \times \frac{\sin 80.91}{\cos 4.12^\circ}$

$= 227.15$ kN

Joint D.



$\tan\theta_3 = \frac{0.3}{5}$

$\theta_3 = 3.43^\circ$

$T_{CD}\cos\theta_2 = T_{DE}\cos\theta_3$

$T_{DE} = \frac{T_{CD}\cos\theta_2}{\cos\theta_3} = \frac{227.15 \times \cos 4.12}{\cos 3.43}$

$= \underline{\underline{226.969 \text{ kN}}}$

Length of the cable.

Segment AC $= \sqrt{5^2 + 0.8^2} = 5.06$ m

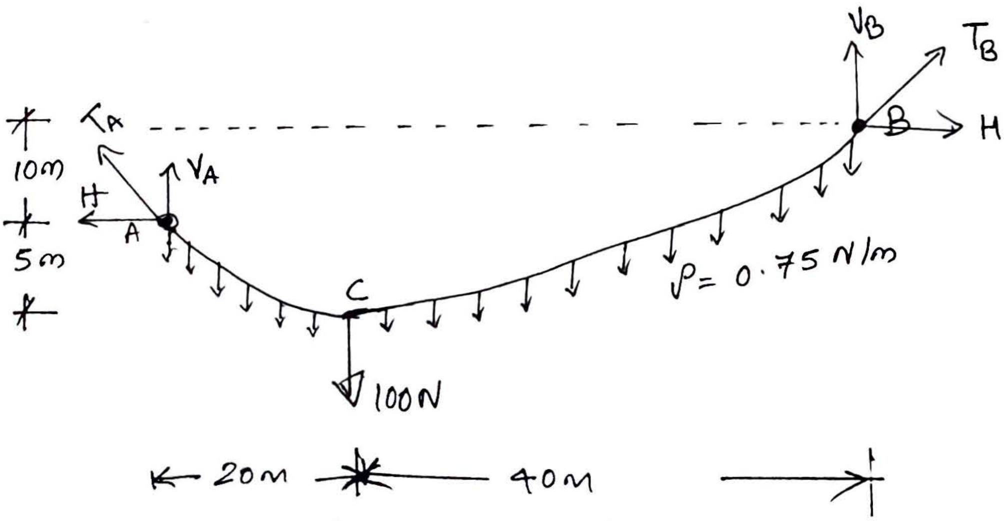
" CD $= \sqrt{5^2 + 0.36^2} = 5.01$ m

" DE $= \sqrt{5^2 + 0.3^2} = 5.01$ m

" EB $= \sqrt{5^2 + 0.86^2} = 5.07$ m

Length of the cable $=$ AC + CD + DE + EB $= \underline{\underline{20.15 \text{ m}}}$

8b.



$L = 20 + 40 = 60$ m

$\Sigma F_V = 0$

$\Rightarrow \quad V_A + V_B = 100 + \frac{3}{4} \times 60 = 145. \quad —(1)$

$\Sigma M_A = 0$

$\Rightarrow$

$$60 \times V_B = 20 \times 100 + \frac{3}{4} \times 60 \times \frac{60}{2} + H \times 10$$

$$V_B = (335 + H)/6 \quad —(2)$$

B.M, $M_C = 0$

$\Rightarrow \quad 40 \times V_B - \frac{3}{4} \times 40 \times 20 - 15H = 0$

$\Rightarrow \quad V_B = 15 + \frac{3}{8} H \quad —(3)$

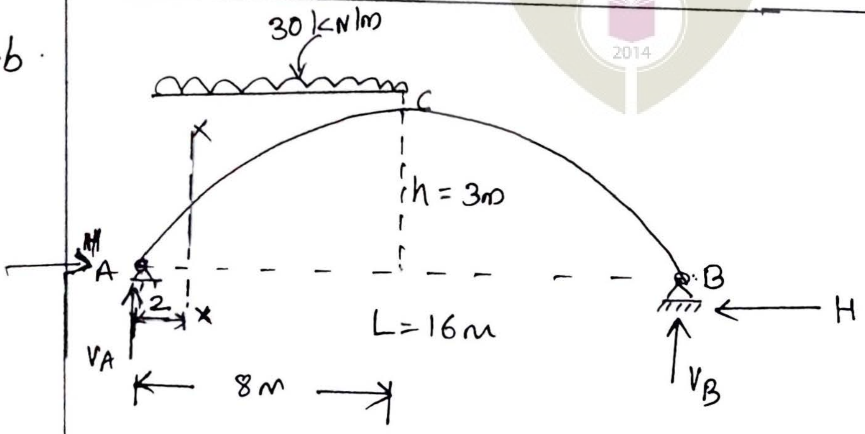
From (2) and (3)

$H = 196\,N$ and $V_B = 88.5N$; $V_A = 56.5\,N$

Since $V_B > V_A$, T_B will be greater than T_A.

$$T_B = \sqrt{V_B^2 + H^2} = \sqrt{88.5^2 + 196^2} = 215\,N$$

9.b.



$V_A + V_B = 30 \times 8 = 240$ kN —(1).

$\Sigma M_A = 0$

$\Rightarrow \quad V_B \times 16 = 30 \times 8 \times 4$

$V_B = 60$ kN; $V_A = 180$ kN

Bending Moment, $M_C = 0$

$\Rightarrow \quad V_A \times 8 - H \times 3 - 30 \times 8 \times 4 = 0$

$180 \times 8 - 3H - 30 \times 32 = 0$

$H = 160 \cdot kN$

Resultant reactions at supports:

@ A, $R_A = \sqrt{V_A^2 + H^2} = \sqrt{180^2 + 160^2} = 240.93$ kN

@ B, $R_B = \sqrt{V_B^2 + H^2} = \sqrt{60^2 + 160^2} = 170.88$ kN

BM, normal thrust & radial shear at 2m from left support.

$x = 2m$ from A.

Equation of parabolic arch:

$$y = \frac{4h}{L^2} \cdot x \cdot (L - x)$$

$$y_{at\, x=2m} = \frac{4 \times 3}{16^2} \times 2(16 - 2) = 1.3125 \text{ m.}$$

$$\frac{dy}{dx}_{at\, x=2m} = \frac{4h}{L^2} \cdot (L - 2x) = \frac{4 \times 3}{16^2}(16 - 2 \times 2) = 0.5625$$

$\Rightarrow \tan\theta = 0.5625$

$\theta_{@\, x=2m} = 29.357^{\circ}$

BM, $M_{x=2} = V_A \times 2 - H \times 1.3125 - 30 \times 2 \times 1$

$= 180 \times 2 - 160 \times 1.3125 - 30 \times 2 \times 1 = 90$ kN-m.

Normal thrust, $T = H\cos\theta + V\sin\theta$

Vertical shear at 2m from A, $V = +V_A = +180$ kN

$T = 160 \cdot \cos 29.36 + 180 \cdot \sin 29.36 = 227.7$ kN

Radial shear $Q = -H\sin\theta + V\cos\theta = -160 \sin 29.36 + 180 \cos 29.36$

$= 78.43$ kN.