Scheme of Valuation/Answer Key

(Scheme of evaluation (marks in brackets) and answers of problems/key)

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 | Duration: 3 Hours

PART A

		Answer any two full questions, each carries 15 marks	Marks
1	a)	$u = sinxcoshy, \quad v = cosxsinhy;$ find u_x, u_y, v_x, v_y $u_x = v_y, \; u_y = -v_x \; ; \quad f'(z) = cosz$	(2+2) (2+1)
	b)	$x = \frac{u}{u^2+v^2} \; , \; y = \frac{-v}{u^2+v^2}$ $\lvert z-2i\rvert = 2 \Rightarrow x^2+y^2-4y = 0$ $\Rightarrow 1+4v = 0.$	(3) (3) (2)
2	a)	Finding $v_x = \frac{2x}{x^2+y^2}+1 \;\; and \;\; v_y = \frac{2y}{x^2+y^2}-2$ C-R equations , $f'(z) = u_x + iv_x$ Put $x = z, y = 0$ $finding \; f(z) = 2i\log z-(2+i)z+C$ OR $u = 2\tan^{-1}\left(\frac{x}{y}\right)-2x-y$ (5) f(z) = u+iv …… (2)	(2) (1+1) (1) (2)
	b)	$u = x^2-y^2, \quad v = 2xy; \quad x = 1 \Rightarrow v^2 = 4(1-u);$ $y = 1 \Rightarrow v^2 = 4(1+u); \quad x+y = 1 \Rightarrow u^2 = 1-2v;$ region.	(2+2) (1+2+1)
3	a)	$f(z) = \frac{zRe(z)}{\lvert z\rvert} = \frac{(x+iy)x}{\sqrt{x^2+y^2}}$ Let $z \to 0$ along the path $y = mx$ $\lim_{z\to 0}\frac{f(z)-f(0)}{z} = \frac{1}{\sqrt{1+m^2}}$, not unique. So limit does not exist.	(2) (2) (2) (1)

		So $f(z)$ is not differentiable at $z = 0$	
	b)	Cross Ratio and Substitution	(4+4)
		PART B	
		Answer any two full questions, each carries 15 marks	
4	a)	$z = 1$ lies inside C and $z = \pm 2i$ lie outside C; $f(z) = \frac{e^z}{z^2+4}$; $\int_c \frac{e^z/(z^2+4)}{(z-1)} dz = 2\pi i \frac{d}{dz}\left(\frac{e^z}{z^2+4}\right)$; $\frac{6e\pi i}{25}$	(2+2) (2+1)
	b)	$(\bar{z})^2 = (x^2 - y^2) - i2xy$ i)Along the real axis, integral = $\frac{8}{3}$, along the vertical line, integral = $2 + \frac{11}{3}i$ Answer : $\frac{14+11i}{3}$ ii) $2y = x$ implies 2dy = dx Answer : $\frac{10-5i}{3}$	(1) (2) (1) (1+3)
5	a)	(a) $f(z) = \frac{z}{3!} - \frac{z^3}{5!} \dots$; $z = 0$ is a removable singularity. (b) Poles $z = (2n+1)\frac{\pi}{2}$, $n = 0, \pm 1, \pm 2, \dots$ Res = -1.	(2+1) (3+1)
	b)	$C: \lvert z \rvert = 1$, $z = e^{i\theta}$, $0 \leq \theta \leq 2\pi$, $sin\theta = \frac{z - z^{-1}}{2i} = \frac{z^2-1}{2iz}$, $d\theta = \frac{dz}{iz}$ $\int_0^{2\pi} \frac{1}{5 - 3sin\theta} d\theta = 2\int \frac{1}{-3z^2 + 10iz + 3} dz$ Poles are $z = 3i, \frac{i}{3}$; $z = 3i$ lies outside C and $z = \frac{i}{3}$ lies inside C. $Resf\left(z = \frac{i}{3}\right) = \frac{1}{8i}$; $\int \frac{1}{-3z^2+10iz+3} dz = \frac{\pi}{4}$; ans $\frac{\pi}{2}$	(2) (2) (1) (1+1+1)
6	a)	$z = e^{i\theta}$,$0 \leq \theta \leq 2\pi$,$dz = ie^{i\theta} d\theta$ $f(z) = logz = i\theta$ $\int_C \log z\, dz = -\int_0^{2\pi} \theta e^{i\theta} d\theta = 2\pi i$	(3) (1) (2+1)
	b)	The curve C consisting of the real axis from $-R$ to R and the upper semicircle $C_R: \lvert z \rvert = R$. As $R \to \infty$, $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx = \int_C \frac{z^2}{(z^2+1)(z^2+4)} dz = \int_C f(z) dz$	(2) (1) (1)

		Poles are $z = \pm i, \pm 2i$ $z = i, 2i$ are the poles of order 1 lying inside C. $Res\ f(z = i) = \frac{i}{6}$, $Res\ f(z = 2i) = \frac{-i}{3}$ By Cauchy's Residue Theorem, $\int_C f(z)dz = 2\pi i(\frac{i}{6} - \frac{i}{3}) = \frac{\pi}{3}$	(1) (2) (1)
PART C			
Answer any two full questions, each carries 20 marks			
7	a)	$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 3 & 2 & 3 \\ 0 & -15 & -10 & -15 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; Rank =2.	(4+3+1)
	b)	$[A:B] = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 1 & 3 & 5 & 9 \\ 2 & 5 & a & b \end{bmatrix}$; *Reducing to Echelon form* $(i)\ a = 8, b \neq 15$ $(ii) a \neq 8, b$ *any real number* $(iii) a = 8, b = 15$	(1+2) (2+1+1)
	c)	Reducing $\begin{bmatrix} 3 & 4 & 0 & 1 \\ 2 & -1 & 3 & 5 \\ 1 & 6 & -8 & -2 \end{bmatrix}$ to row echelon form. Rank = 3 = maximum number of linearly independent vectors Given vectors are linearly independent.	(2) (2) (1)
8	a)	Reduce the Augmented matrix into Echelon form. $x = 2, y = 1, z = -4$	(5+3)
	b)	$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ $\lambda^3 - 6\lambda^2 + 8\lambda + 2 = 0$	(3+3)
	c)	Eigen values = 4,1,7; eigen vectors [-1 2 2],[2 -1 2],[2 2 -1]	(3+3)
9	a)	Characteristic equation $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$; $\lambda = 1,1,4$ $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}$	(2) (2) (4)
	b)	Definition of symmetric and skew symmetric matrix; $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$ Proving $A + A^T$ is symmetric *and* $A - A^T$ is skew symmetric.	(3+3)

	c)	$A = \begin{bmatrix} 3 & 11 \\ 11 & 3 \end{bmatrix}$; Eigen values are 14,-8 ; canonical form $14x^2-8y^2=0$. Pair of Straight Line	(2+2+1+1)
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