Reg No.:_______________ Name:__________________________

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

THIRD SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2019

**Course Code: MA201**

**Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS**

Max. Marks: 100 Duration: 3 Hours

## PART A

***Answer any two full questions, each carries 15 marks*** Marks

1 a) Prove that the function $sinz$ is analytic and find its derivative. (7)

b) Under the transformation $w = \frac{1}{z}$ , find the image of $|z - 2i| = 2$ (8)

2 a) Find the analytic function whose imaginary part is (7)

$v(x, y) = log(x^2 + y^2) + x - 2y.$

b) Under the transformation $w = z^2$, find the image of the triangular region bounded by $x = 1,\ y = 1$ and $x + y = 1$. (8)

3 a) Show that $f(z) = \begin{cases} \frac{zRe(z)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not differentiable at $z = 0$ (7)

b) Find the bilinear transformation that maps the points $-1, i, -1$ onto $i, 0, -i$. (8)

## PART B

***Answer any two full questions, each carries 15 marks***

4 a) Using Cauchy's integral formula, evaluate $\int_C \frac{e^z}{(z^2+4)(z-1)^2}\, dz$ , where $C$ is the circle $|z - 1| = 2$ . (7)

b) Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along (8)

(i) the real axis to 2 and then vertically to $2 + i$ .

ii) the line $2y = x$

5 a) Find all singular points and residues of the functions (7)

(a) $f(z) = \frac{z-sinz}{z^2}$ $(b)$ $f(z) = tan\, z$

b) Evaluate $\int_0^{2\pi} \frac{1}{5-3sin\theta} d\theta$ . (8)

6 a) Evaluate $\int_C \, logzdz$ where $C$ is the circle $|z| = 1$ (7)

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ (8)

## PART C

***Answer any two full questions, each carries 20 marks***

7 a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$ (8)

b) Find the values of $a$ and $b$ for which the system of linear equations (7)

$x + 2y + 3z = 6, x + 3y + 5z = 9, 2x + 5y + az = b$ has (i) no solution

(ii) a unique solution (iii) infinitely many solutions

c) Show that the vectors $[3 \quad 4 \quad 0 \quad 1]$ , $[2 \quad -1 \quad 3 \quad 5]$ and $[1 \quad 6 \quad -8 \quad -2]$ are linearly independent in $R^4$. (5)

8 a) Solve the system of equations by Gauss Elimination Method: (8)

$3x + 3y + 2z = 1, \qquad x + 2y = 4, 10y + 3z = -2, \qquad 2x - 3y - z = 5$

b) Find the nature, index, rank and signature of the quadratic form (6)

$x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_1x_3 + 2x_2x_3$

c) Find the Eigen values and Eigen vectors of $\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$ (6)

9 a) Diagonalize the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (8)

b) Define symmetric and skew symmetric matrices. Show that any real square matrix can be written as the sum of a symmetric and a skew symmetric matrix. (6)

c) What type of conic section is represented by the quadratic form (6)

$3x^2+22xy+3y^2 = 0$ by reducing it into canonical form.

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