

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**SECOND SEMESTER B.TECH DEGREE EXAMINATION (R & S), MAY 2019**

**Course Code: MA102**

**Course Name: DIFFERENTIAL EQUATIONS**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks*

- 1 Find the general solution of  $\frac{d^3 y}{dx^3} + y = 0$  (3)
- 2 Find the Wronskian of  $e^x \cos 2x$  and  $e^x \sin 2x$  (3)
- 3 Find the Particular Integral of  $y'' - 4y' - 5y = 4 \cos 2x$ . (3)
- 4 Find the particular integral of  $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 4y = \sinh 2x$  (3)
- 5 Evaluate the coefficient  $a_n$  in the Fourier series expansion for  $f(x) = |\sin x|$  in  $-\pi < x < \pi$  (3)
- 6 Find the half range Fourier sine series representation of  $f(x) = k$  in  $(0, \pi)$  (3)
- 7 Find the partial differential equation of all spheres having their centre lies on z-axis. (3)
- 8 Form the partial differential equation of  $z = f\left(\frac{xy}{z}\right)$  by eliminating the arbitrary function f. (3)
- 9 Solve  $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$ ,  $u(0, y) = 8e^{-3y}$ , using the method of separation of variables. (3)
- 10 A tightly stretched string of length  $l$  is fixed at both ends and pulled from its mid point to a height  $h$  and released from rest from this position. Write down the initial and boundary conditions. (3)
- 11 Find the steady state temperature distribution in a rod of length 30 cm, if the ends of the rod are kept at  $20^\circ C$  and  $80^\circ C$  (3)
- 12 Write down the three possible solutions of the one dimensional heat equation. (3)

**PART B***Answer six questions, one full question from each module***Module 1**

- 13 a) Solve the initial value problem  $y'' + 4y' + 5y = 0, y(0) = 2, y'(0) = -5$ . (6)
- b) Find the general solution of the differential equation  $y''' - y'' + 4y' = 0$  (5)

**OR**

- 14 a) If  $y_1(x) = x$  is a solution to the differential equation  
 $(1 + x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$ , find the general solution. (6)
- b) Solve the ordinary differential equation  $y''' - 3y'' - 4y' + 6y = 0$ . (5)

**Module 1I**

- 15 a) Solve  $2(3x + 1)^2 \frac{d^2y}{dx^2} + 21(3x + 1) \frac{dy}{dx} + 18y = 9x$  (6)
- b) Solve  $(D^4 + 2D^2 + 1)y = x^4$  (5)

**OR**

- 16 a) Use method variation of parameters to solve  $\frac{d^2y}{dx^2} + 4y = \tan 2x$  (6)
- b) Solve  $(D^2 - 4D + 4)y = \sin^2 x$  (5)

**Module 1II**

- 17 a) Obtain the half range Fourier cosine series expansion of  $f(x) = x \sin x$  in  $(0, \pi)$ . (6)
- b) Find the Fourier series for  $f(x) = |x|, -\pi < x < \pi$  (5)

**OR**

- 18 a) Find the Fourier series for  $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi, & 0 < x < \pi \end{cases}$  (6)
- b) Find the Fourier series of the periodic function  $f(x)$  of period 4, where  

$$f(x) = \begin{cases} 0, & -2 < x \leq -1 \\ k, & -1 < x < 1 \\ 0, & 1 \leq x < 2 \end{cases}$$
 (5)

**Module 1V**

- 19 a) Solve  $\frac{y^2 z}{x} p + xzq = y^2$  (6)
- b) Find the partial differential equation of all planes which are at a constant distance  $k$  from the origin. (5)

**OR**

- 20 a) Solve  $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$  (6)

b) Solve  $(D^2 + 3DD' + 2D'^2)z = x^2y^2$  (5)

**Module V**

- 21 A string is stretched between two fixed points at a distance of 60 cm and the points of the string are given initial velocities where

$$v = \begin{cases} \frac{\lambda x}{30}, & 0 < x < 30 \\ \frac{\lambda}{30}(60 - x), & 30 < x < 60 \end{cases}, \text{ x being the distance from an end, find the} \quad (10)$$

displacement at any time t.

**OR**

- 22 A uniform elastic string of length 60 cm is subjected to a constant tension of 2 Kg. If the ends are fixed, the initial displacement  $u(x, 0) = 60x - x^2, 0 < x < 60$  and the initial velocity is zero, find the displacement function  $u(x, t)$  (10)

**Module VI**

- 23 Find the temperature distribution in a rod of length 2m whose end points are maintained at temperature  $0^\circ\text{C}$  and the initial temperature is  $f(x) = 100(2x - x^2), 0 \leq x \leq 2$  (10)

**OR**

- 24 A bar 10 cm long with insulated sides has its ends A and B maintained at  $50^\circ\text{C}$  and  $100^\circ\text{C}$  respectively until steady state conditions prevail. The temperature of A is suddenly raised to  $90^\circ\text{C}$  and at the same time that at B is lowered to  $60^\circ\text{C}$ . Find the temperature distribution in the bar at time t. (10)

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